

Calculating and Displaying Fatigue Results

The ANSYS Fatigue Module has a wide range of features for performing calculations and presenting analysis results.

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Introduction

While many parts may work well initially, they often fail in service due to fatigue failure caused by repeated cyclic loading. Characterizing the capability of a material to survive the many cycles a component may experience during its lifetime is the aim of fatigue analysis. In a general sense, Fatigue Analysis has three main methods, Strain Life, Stress Life, and Fracture Mechanics; the first two being available within the ANSYS Fatigue Module.

The Strain Life approach is widely used at present. Strain can be directly measured and has been shown to be an excellent quantity for characterizing low-cycle fatigue. Strain Life is typically concerned with crack initiation, whereas Stress Life is concerned with total life and does not distinguish between initiation and propagation. In terms of cycles, Strain Life typically deals with a relatively low number of cycles and therefore addresses Low Cycle Fatigue (LCF), but works with high numbers of cycles as well. Low Cycle Fatigue usually refers to fewer than 10^5 cycles. Stress Life is based on S-N curves (Stress – Cycle curves) and has traditionally dealt with relatively high numbers of cycles and therefore addresses High Cycle Fatigue (HCF), greater than 10^5 cycles inclusive of infinite life.

Fracture Mechanics starts with an assumed flaw of known size and determines the crack's growth as is therefore sometimes referred to as "Crack Life". Fracture Mechanics is widely used to determine inspection intervals. For a given inspection technique, the smallest detectable flaw size is known. From this detectable flaw size we can calculate the time required for the crack to grow to a critical size. We can then determine our inspection interval to be less than the crack growth time. Sometimes, Strain Life methods are used to determine crack initiation with Fracture Mechanics used to determine the crack life. In this situation, crack initiation plus crack life equals the total life of the part.

Analysis Decisions

Common Decisions for Fatigue Analysis

There are 5 common input decision topics upon which your fatigue results are dependent upon. These fatigue decisions are grouped into the types listed below:

- Fatigue Analysis Type
- Loading Type
- Mean Stress Effects
- Multiaxial Stress Correction
- Fatigue Modification Factor

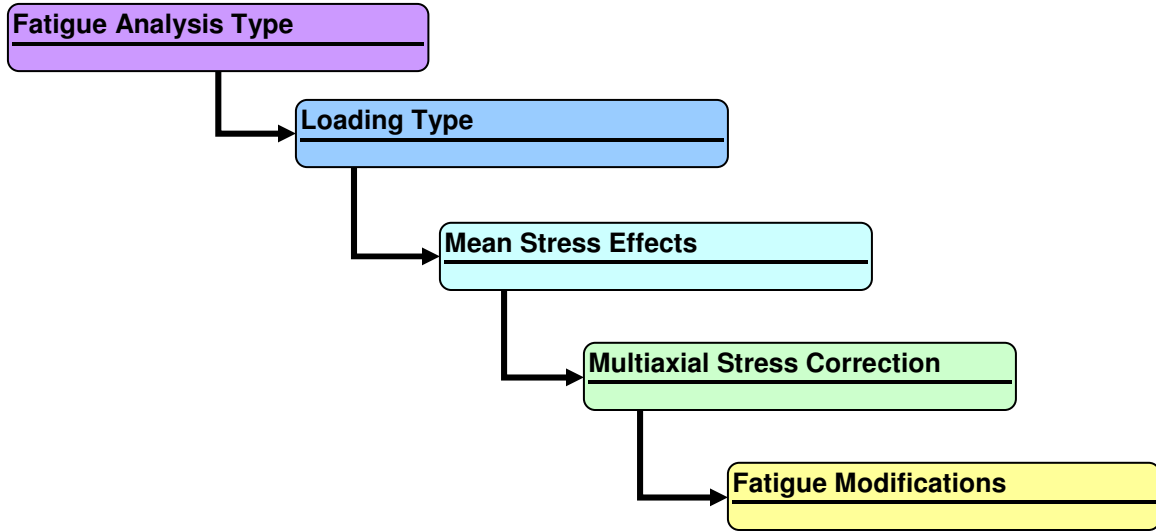


Figure 1. Simplified Fatigue Analysis Decision Tree

The decision tree in Figure 1 above shows the general flow of decision required to perform a fatigue analysis. We will now explore each one in detail.

Stress Life vs. Strain Life

Within the ANSYS fatigue module, the first decision that needs to be made in performing a fatigue analysis is which type of fatigue analysis to perform – Stress Life or Strain Life. Stress Life is based on empirical S-N curves and then modified by a variety of factors. Strain Life is based upon the Strain Life Relation Equation where the Strain Life Parameters are values for a particular material that best fit the equation to measured results. The Strain Life Relation requires a total of 6 parameters to define the strain-life material properties; four strain-life parameter properties and the two cyclic stress-strain parameters. The Strain Life Relation equation is shown below:

$$\frac{\Delta \epsilon}{2} = \frac{\sigma_f'}{E} (2N_f)^b + \epsilon_f' (2N_f)^c$$

The two cyclic stress-strain parameters are part of the equation below:

$$\Delta \epsilon = \frac{\Delta \sigma}{E} + 2 \left(\frac{\Delta \sigma}{2K'} \right)^{1/n'}$$

Where:

$$\frac{\Delta \epsilon}{2} = \text{Total Strain Amplitude}$$

$$\Delta \sigma = 2 \times \text{the Stress Amplitude}$$

$$E = \text{Modulus of Elasticity}$$

$$N_f = \text{Number of Cycles to Failure}$$

$$2N_f = \text{Number of Reversals to Failure}$$

And the parameters required for a Strain Life analysis are:

$$\sigma'_f = \text{Fatigue Strength Coefficient}$$

$$B = \text{Fatigue Strength Exponent (Basquin's Exponent)}$$

$$\epsilon'_f = \text{Fatigue Ductility Coefficient}$$

$$C = \text{Fatigue Ductility Exponent}$$

$$K' = \text{Cyclic Strength Coefficient}$$

$$n' = \text{Cyclic Strain Hardening Exponent}$$

Note that in the above equation, total strain (elastic + plastic) is the required input. However, running an FE analysis to determine the total response can be very expensive and wasteful, especially if the nominal response of the structure is elastic. An accepted approach is to assume a nominally elastic response and then make use of Neuber's equation to relate local stress/strain to nominal stress/strain at a stress concentration location.

To relate strain to stress we use Neuber's Rule, which is shown below:

$$\epsilon \sigma = K_t^2 e S$$

Where:

$$\epsilon = \text{Local (Total) Strain}$$

$$\sigma = \text{Local Stress}$$

$$K_t = \text{Elastic Stress Concentration Factor}$$

$$e = \text{Nominal Elastic Strain}$$

$$S = \text{Nominal Elastic Stress}$$

Thus by simultaneously solving Neuber's equation along with cyclic strain equation, we can thus calculate the local stress/strains (including plastic response) given only elastic input. Note that this calculation is nonlinear and is solved via iterative methods. Also note that ANSYS fatigue uses a value of 1 for K_t , assuming that the mesh is refined enough to capture any stress concentration effects. This K_t is not to be confused with the Stress Reduction Factor option which is typically used in Stress life analysis to account for things such as reliability and size effects.

Common Decisions to Both Types of Fatigue Analysis

Once the decision on which type of fatigue analysis to perform, Stress Life or Strain Life, there are 4 other topics upon which your fatigue results are dependent upon. Input decisions that are common to both types of fatigue analyses are listed below:

- Loading Type
- Mean Stress Effects
- Multiaxial Stress Correction
- Fatigue Modification Factor

Within Mean Stress Effects, the available options are quite different. In the following sections, we will explore all of these additional decisions. These input decision trees for both Stress Life and Strain Life are outlined in Figures 1 and 2.

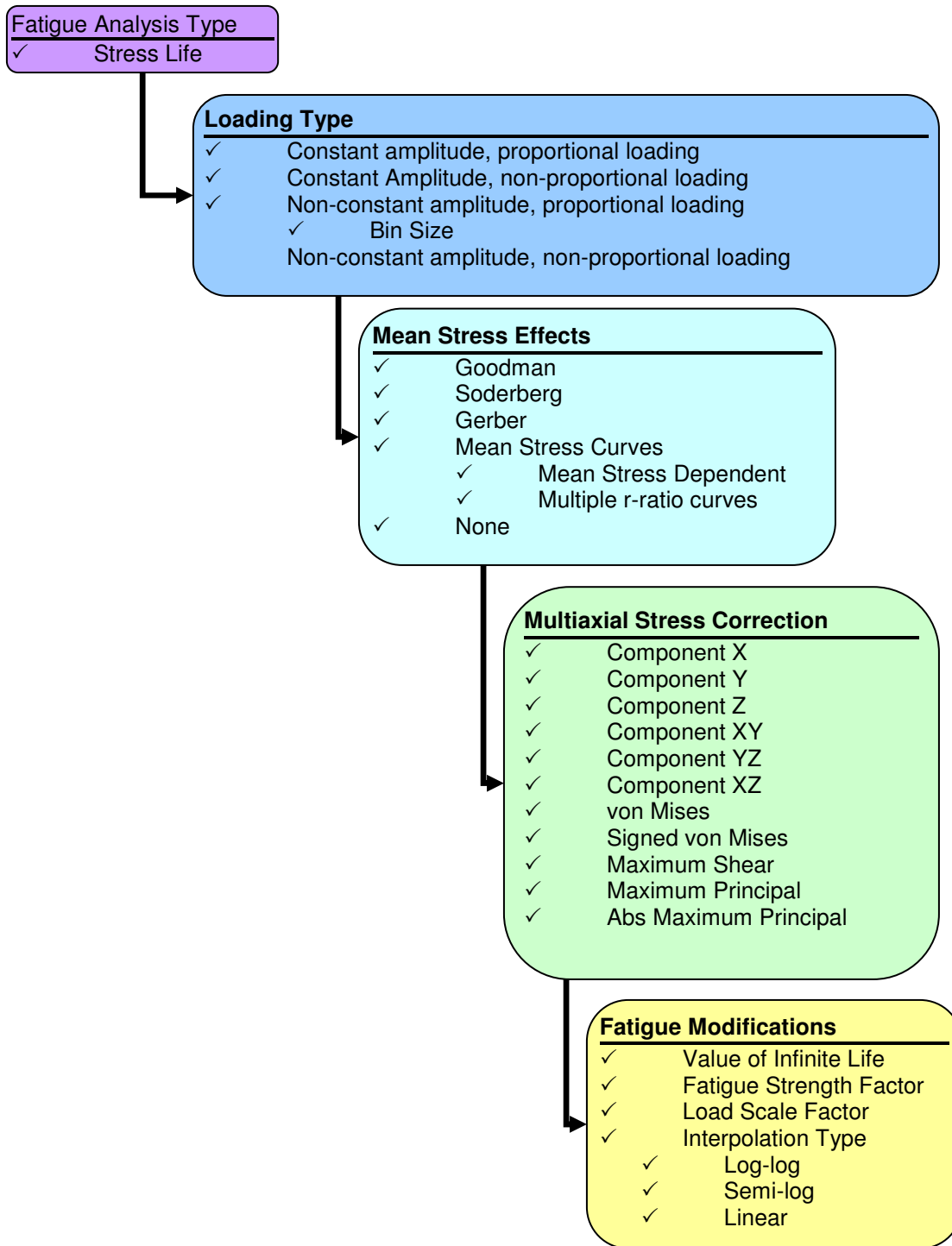


Figure 2. Stress Life Decision Tree. ✓ Indicates capability available in the ANSYS Fatigue Module

As can be seen in the Stress Life Decision Tree, we need to make four input decisions to perform a Stress Life analysis. These decisions affect the outcome of the fatigue analysis in both predicted life and types of post processing available.

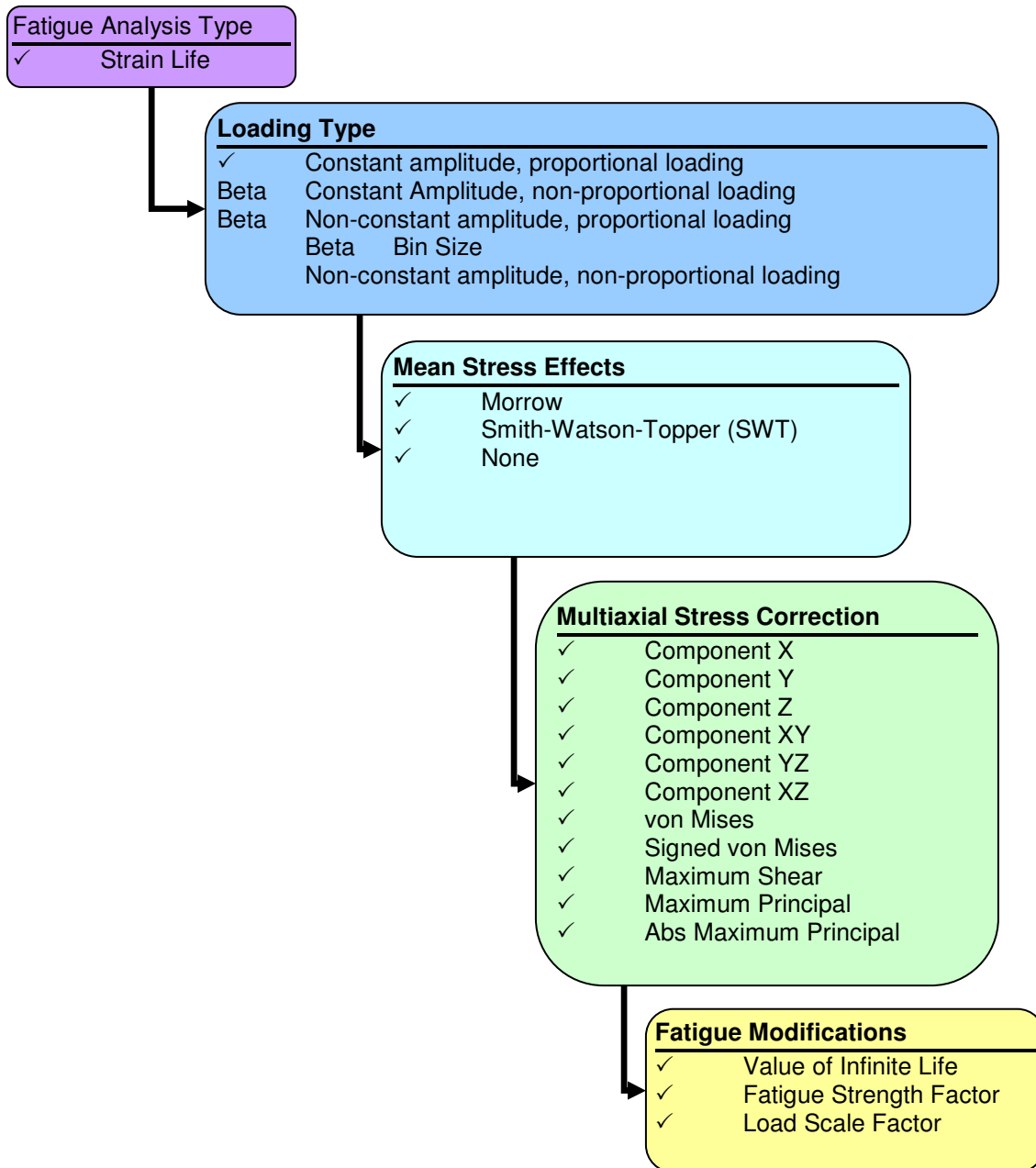


Figure 3. Strain Life Decision Tree. ✓ Indicates capability available in the ANSYS Fatigue Module. Beta indicates a beta capability available in the ANSYS Fatigue Module.

Very similar to the Stress Life Decision Tree, we can see that we also need to make four input decisions to perform a Strain Life fatigue analysis. These decisions affect the outcome of the fatigue analysis in both predicted life and types of post processing available. We will look at each of these choices in detail below.

Types of Cyclic Loading

Unlike static stress, which is analyzed with calculations for a single stress state, fatigue damage occurs when stress at a point changes over time. There are essentially four classes of fatigue loading, with the ANSYS Fatigue Module currently supporting the first three:

- Constant amplitude, proportional loading
- Constant amplitude, non-proportional loading
- Non-constant amplitude, proportional loading
- Non-constant amplitude, non-proportional loading

In the above descriptions, the amplitude identifier is readily understood. Is the loading a variant of a sine wave with a single load ratio or does the loading vary perhaps erratically, with the load ratio changing with time? The second identifier, proportionality, describes whether the changing load causes the principal stress axes to change. If the principal stress axes do not change, then it is proportional loading. If the principal stress axes do change, then the cycles cannot be counted simply and it is non-proportional loading.

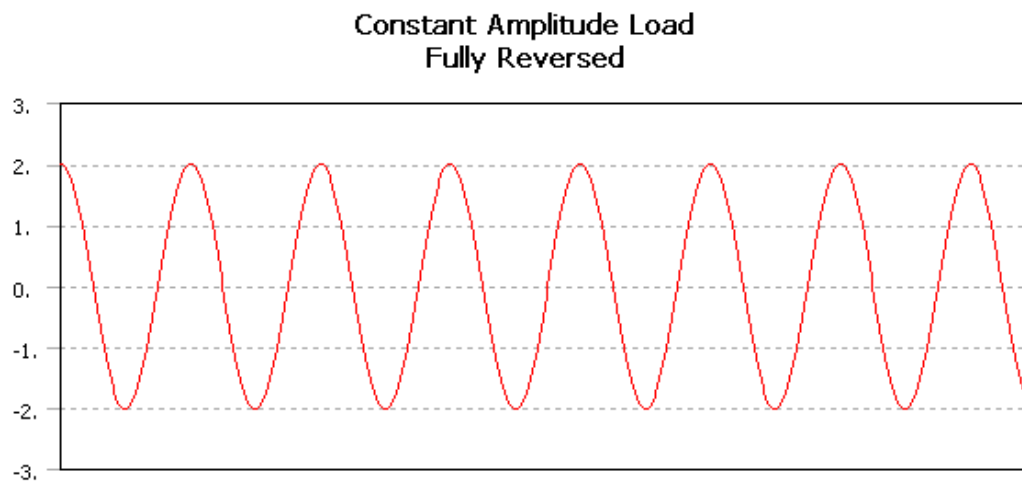


Figure 4. Example of constant amplitude loading. In this case it is fully reversed (+1 to -1) loading.

Constant amplitude, Proportional Loading

Constant amplitude, proportional loading is the classic, “back of the envelope” calculation describing whether the load has a constant maximum value or continually varies with time. Loading is of constant amplitude because only one set of FE stress results along with a loading ratio is required to calculate the alternating and mean values. The loading ratio is defined as the ratio of the second load to the first load ($LR = L_2/L_1$). Loading is proportional since only one set of FE results are needed (principal stress axes do not change over time). Common types of constant amplitude loading are fully reversed (apply a load, then apply an equal and opposite load; a load ratio of -1) and zero-based (apply a load then remove it; a load ratio of 0). Since loading is proportional, looking at a single set of FE results can identify critical fatigue locations. Likewise, since there are only two loadings, no cycle counting or cumulative damage calculations need to be done.

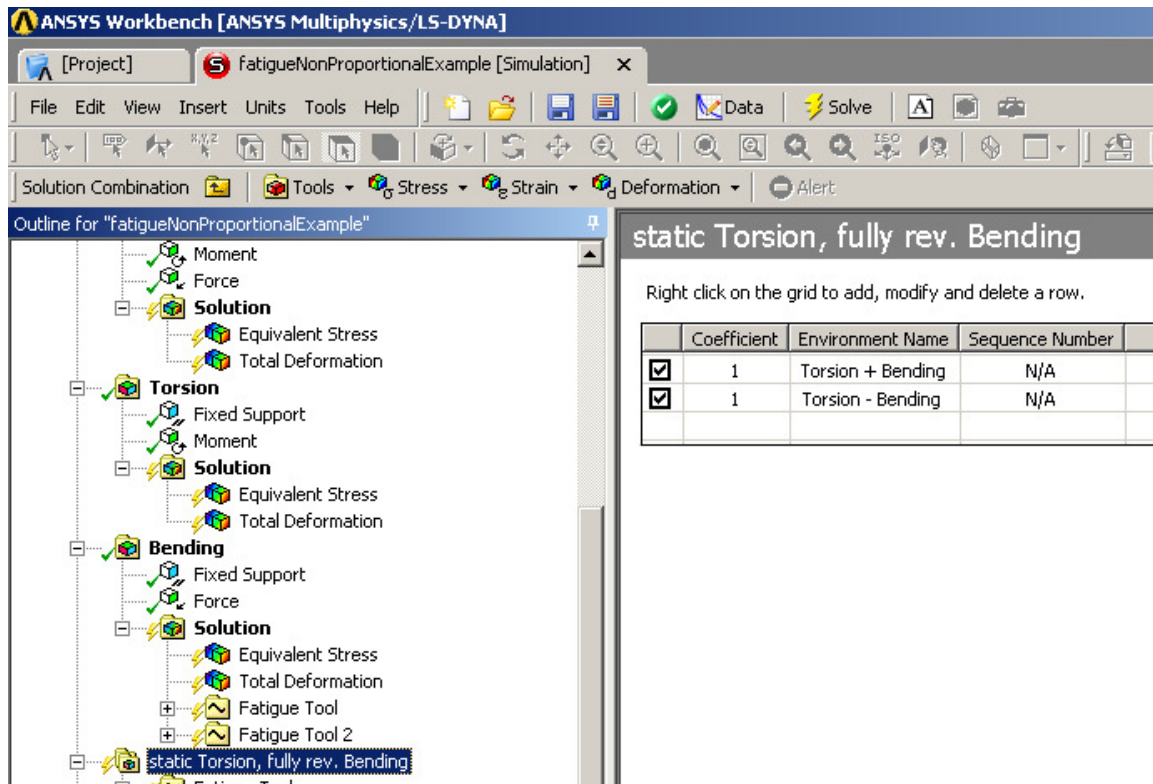


Figure 5. Example tree and solution combination for Constant Amplitude, non-proportional loading.

Constant Amplitude, Non-Proportional Loading

Constant Amplitude, non-proportional loading looks at exactly two load cases that need not be related by a scale factor. The loading is of constant amplitude but non-proportional since principal stress or strain axes are free to change between the two load sets. No cycle counting needs to be done. But since the loading is non-proportional, the critical fatigue location may occur at a spatial location that is not easily identifiable by looking at either of the base loading stress states. This type of fatigue loading can describe common fatigue loadings such as:

- Alternating between two distinct load cases (like a bending load and torsional load)
- Applying an alternating load superimposed on a static load.
- Analyses where loading is proportional but results are not. This happens under conditions where changing the direction or magnitude of loads causes a change in the relative stress distribution in the model. This may be important in situations with nonlinear contact, compression-only surfaces, or bolt loads.

Fatigue tools located under a solution branch are inherently applied to that single branch and thus can only handle proportional loading. In order to handle non-proportional loading, the fatigue tool must be able to span multiple solutions. This is accomplished by adding a fatigue tool under the solution combination folder that can indeed span multiple solution branches.

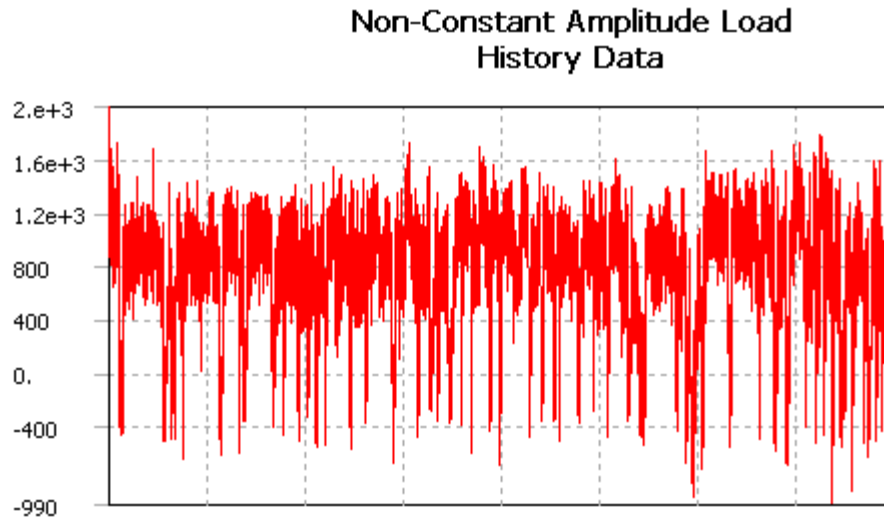


Figure 6. Example of Non-constant amplitude loading.

Non-constant amplitude, Proportional Loading

Non-constant amplitude, proportional loading also needs only one set of FE results. But instead of using a single load ratio to calculate alternating and mean values, the load ratio varies over time. Think of this as coupling an FE analysis with strain-gauge results collected over a given time interval. Since loading is proportional, the critical fatigue location can be found by looking at a single set of FE results. However, the fatigue loading which causes the maximum damage cannot easily be seen. Thus, cumulative damage calculations (including cycle counting such as Rainflow and damage summation such as Miner's rule) need to be done to determine the total amount of fatigue damage and which cycle combinations cause that damage. Cycle counting is a means to reduce a complex load history into a number of events, which can be compared to the available constant amplitude test data.

Non-constant Amplitude, proportional loading within the ANSYS Fatigue Module uses a "quick counting" technique to substantially reduce runtime and memory. In quick counting, alternating and mean stresses are sorted into bins before partial damage is calculated. Without quick counting, data is not sorted into bins until after partial damages are found. The accuracy of quick counting is usually very good if a proper number of bins are used when counting. The bin size defines how many divisions the cycle counting history should be organized into for the history data loading type. Strictly speaking, bin size specifies the number of divisions of the rainflow matrix. A larger bin size has greater precision but will take longer to solve and use more memory. Bin size defaults to 32, meaning that the Rainflow Matrix is 32 x 32 in dimension.

For Stress Life, another available option when conducting a variable amplitude fatigue analysis is the ability to set the value used for infinite life. In constant amplitude loading, if the alternating stress is lower than the lowest alternating stress on the fatigue curve, the fatigue tool will use the life at the last point. This provides for an added level of safety because many materials do not exhibit an endurance limit. However, in non-constant amplitude loading, cycles with very small alternating stresses may be present and may incorrectly predict too much damage if the number of the small stress cycles is high enough. To help control this, the user can set the infinite life value that will be used if the alternating stress is beyond the limit of the SN curve. Setting a higher value will make small stress cycles less damaging if they occur many times. The Rainflow and damage matrix results can be helpful in determining the effects of small stress cycles in your loading history.

Non-constant amplitude, Non-Proportional Loading

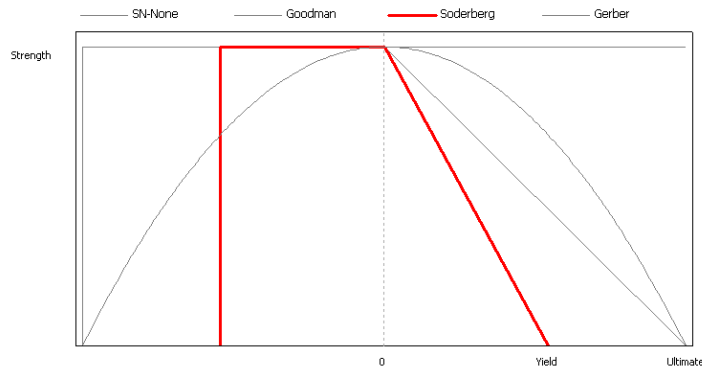
Non-constant amplitude, non-proportional loading is the most general case and is similar to Constant Amplitude, non-proportional loading, but in this loading class there are more than 2 different stress cases involved that have no relation to one another. Not only is the spatial location of critical fatigue life unknown, but also unknown is what combination of loads cause the most damage. Thus, more advanced cycle counting is required such as path independent peak methods or multiaxial critical plane methods. Currently the ANSYS Fatigue Module does not support this type of fatigue loading.

Mean Stress Correction

Once you have made the decision on which type of fatigue analysis to perform, Stress Life or Strain Life, and have determined your loading type, the next decision is whether to apply a mean stress correction. Cyclic fatigue properties of a material are often obtained from completely reversed, constant amplitude tests. Actual components seldom experience this pure type of loading, since some mean stress is usually present. If the loading is other than fully reversed, a mean stress exists and may be accounted for.

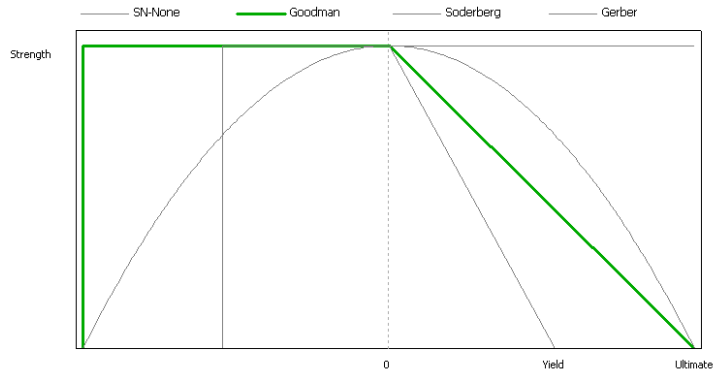
Mean Stress Corrections for Stress Life

For Stress Life, if experimental data at different mean stresses or r-ratio's exist, mean stress can be accounted for directly through interpolation between material curves. If experimental data is not available, several empirical options may be chosen including Gerber, Goodman and Soderberg theories which use static material properties (yield stress, tensile strength) along with S-N data to account for any mean stress.



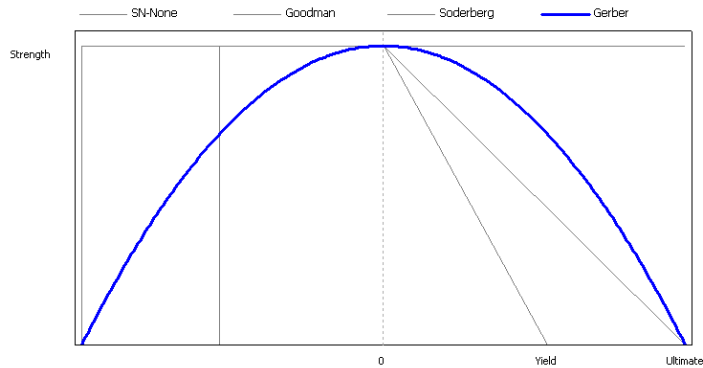
$$\frac{\sigma_{Alternating}}{S_{Endurance_Limit}} + \frac{\sigma_{Mean}}{S_{Yield_Strength}} = 1 \quad \text{Soderberg Equation}$$

Figure 7. Equation and graphical representation of the Soderberg Mean Stress Correction for Stress Life Fatigue Analysis.



$$\frac{\sigma_{\text{Alternating}}}{S_{\text{Endurance_Limit}}} + \frac{\sigma_{\text{Mean}}}{S_{\text{Ultimate_Strength}}} = 1 \quad \text{Goodman Equation}$$

Figure 8. Equation and graphical representation of the Goodman Mean Stress Correction for Stress Life Fatigue Analysis.



$$\frac{\sigma_{\text{Alternating}}}{S_{\text{Endurance_Limit}}} + \left(\frac{\sigma_{\text{Mean}}}{S_{\text{Ultimate_Strength}}} \right)^2 = 1 \quad \text{Gerber Equation}$$

Figure 9. Equation and graphical representation of the Gerber Mean Stress Correction for Stress Life Fatigue Analysis.

In general, most experimental data fall between the Goodman and Gerber theories with the Soderberg theory usually being overly conservative. The Goodman theory can be a good choice for brittle materials with the Gerber theory usually a good choice for ductile materials. The Gerber theory treats negative and positive mean stresses the same whereas Goodman and Soderberg are not bounded when using negative mean stresses. Therefore, within the ANSYS fatigue module the alternating stress is capped by ignoring the negative mean stress. Additionally the negative mean stress is capped to either the yield stress or the ultimate stress for Soderberg and Goodman respectively. See Figures 6 and 7 for clarification. Goodman and Soderberg are conservation approaches because although a compressive mean stress can retard fatigue crack

growth, ignoring a negative mean is usually more conservative. Of course, the option of no mean stress correction is also available.

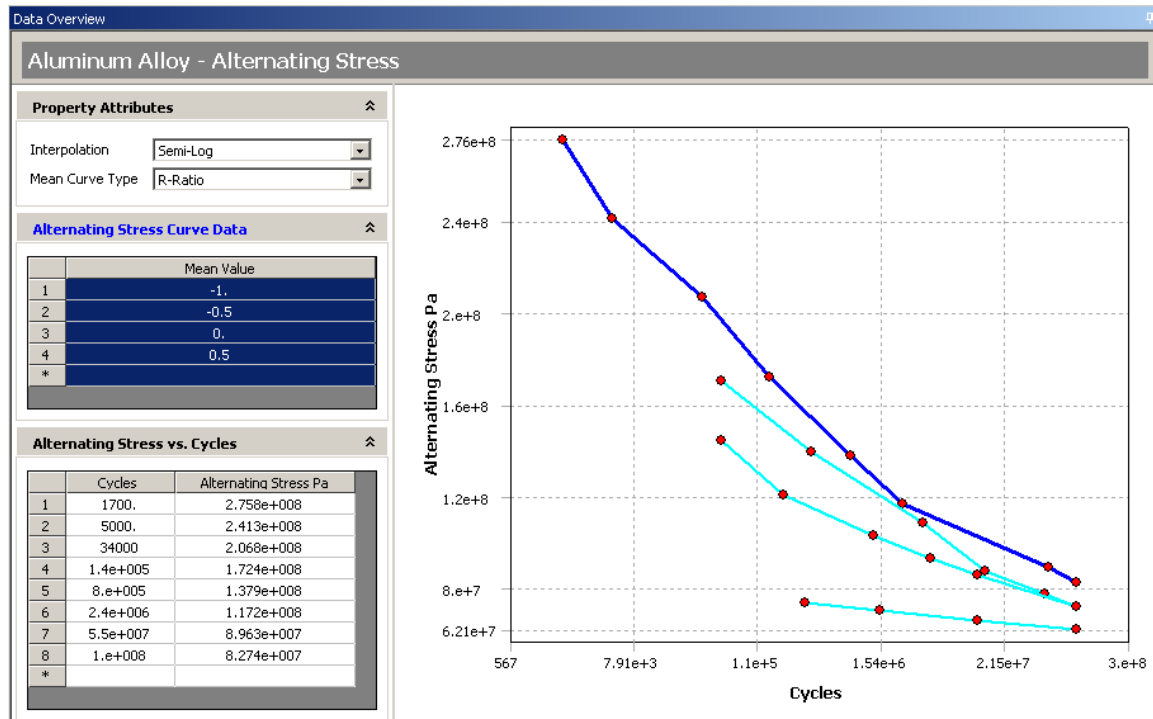


Figure 10. Example of the Mean Stress Correction data by r-ratio mean stress curves.

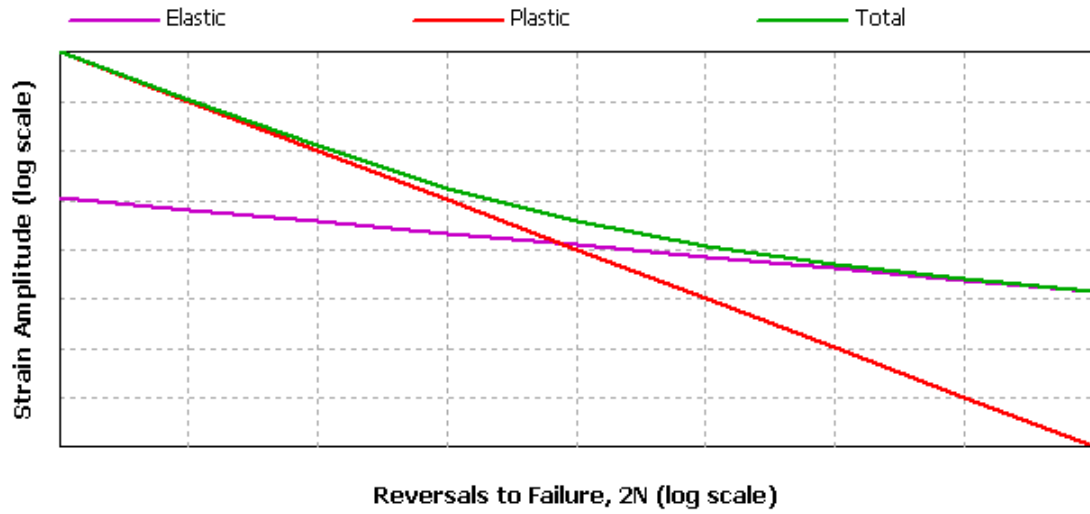
A fifth mean stress correction is by empirical data, which is the selection “Mean Stress Curves” in the Fatigue Details View. Mean Stress Curves uses experimental fatigue data to account for mean stress effects. There are two types of stress curves available, mean value stress curves and r-ratio stress curves. For mean stress value curves, the testing apparatus applied a constant mean stress while applying a varying alternating stress. In practice, this is relatively hard to do. R-ratio stress curves are similar to the mean value stress curves, but instead of maintaining a particular mean stress, the testing apparatus applies a consistent loading ratio. This is typically easier to perform in actual practice.

The loading ratio is defined as the ratio of the second load to the first load ($LR = L_2/L_1$). Loading is proportional since only one set of FE results are needed (principal stress axes do not change over time). Common types of constant amplitude loading are fully reversed (apply a load, then apply an equal and opposite load; a load ratio of -1) and zero-based (apply a load then remove it; a load ratio of 0).

Note that if an empirical mean stress theory is chosen, such as Goodman, and multiple SN curves are defined, any mean stresses that may exist will be ignored when querying the material data since an empirical theory was chosen. Thus if you have multiple r-ratio SN curves and use the Goodman theory, the SN curve at $r=-1$ will be used. In general, it is not advisable to use an empirical mean stress theory if multiple mean stress data exists.

Mean Stress Corrections for Strain Life

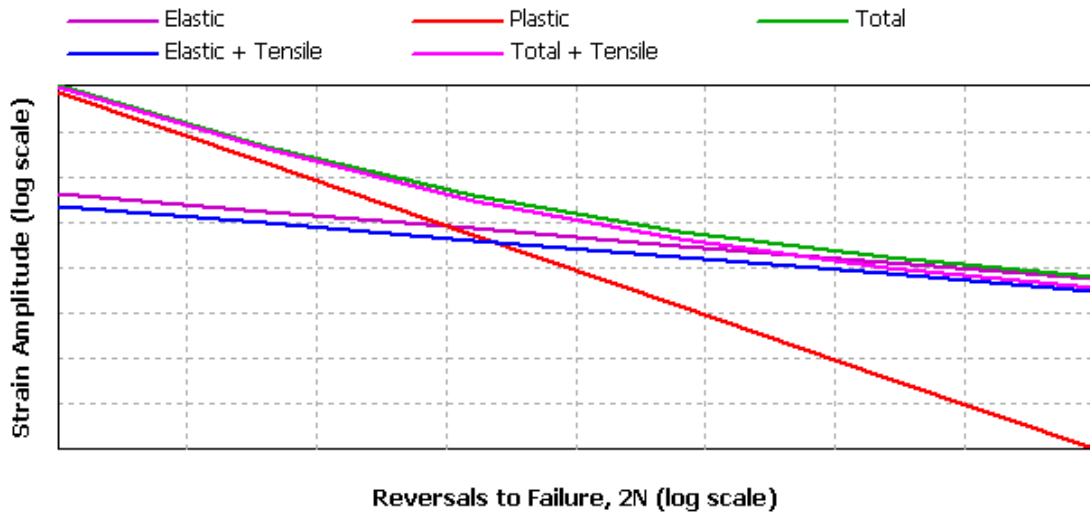
For Strain Life, the ANSYS Fatigue Module has a variety of mean stress correction methods including no mean stress effects, Morrow and Smith-Watson-Topper (SWT).



$$\frac{\Delta \epsilon}{2} = \frac{\sigma'_{failure}}{E} (2N_{failure})^b + \epsilon'_{failure} (2N_{failure})^c$$

Basic Strain Life Relation Equation

Figure 11. Example of the No Mean Stress Correction for Strain Life Fatigue Analysis.

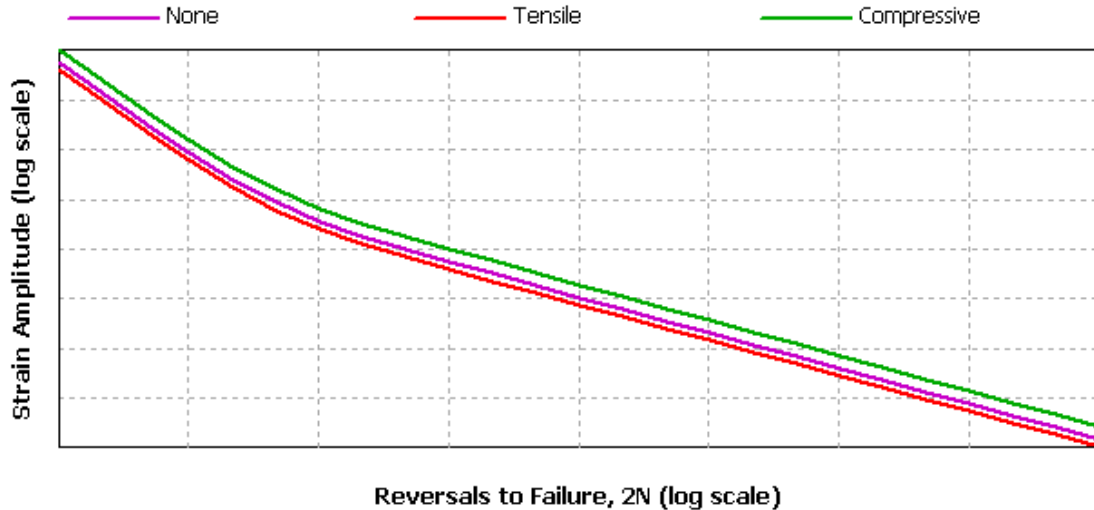


$$\frac{\Delta \epsilon}{2} = \frac{\sigma'_{failure} - \sigma_{Mean}}{E} (2N_{failure})^b + \epsilon'_{failure} (2N_{failure})^c$$

Strain Life Equation accounting for Morrow Mean Stress Correction

Figure 12. Example of the Morrow Mean Stress Correction for Strain Life Fatigue Analysis.

In Morrow's method, the elastic term in the strain-life equation is modified by the mean stress. This modification is consistent with observations that the mean stress effects are significant at low values of plastic strain, where elastic strain dominates, and that mean stress has little effect at shorter life, where plastic strains dominate. Unfortunately, it incorrectly predicts that the ratio of elastic to plastic strain is dependent on mean stress, which is not true.



$$\sigma'_{Maximum} \frac{\Delta \epsilon}{2} = \frac{(\sigma'_{failure})^2}{E} (2N_{failure})^{2b} + \sigma'_{failure} \epsilon'_{failure} (2N_{failure})^{b+c}$$

Strain Life Equation accounting for Smith, Watson and Topper Mean Stress Correction

Figure 13. Example of the Smith, Watson and Topper (SWT) Mean Stress Correction for Strain Life Fatigue Analysis.

Smith, Watson and Topper (SWT) suggested a different equation to account for the presence of mean stresses. It has the limitation that it is undefined for negative maximum stresses. The physical interpretation of this is that no fatigue damage occurs unless tension is present at some point during the loading.

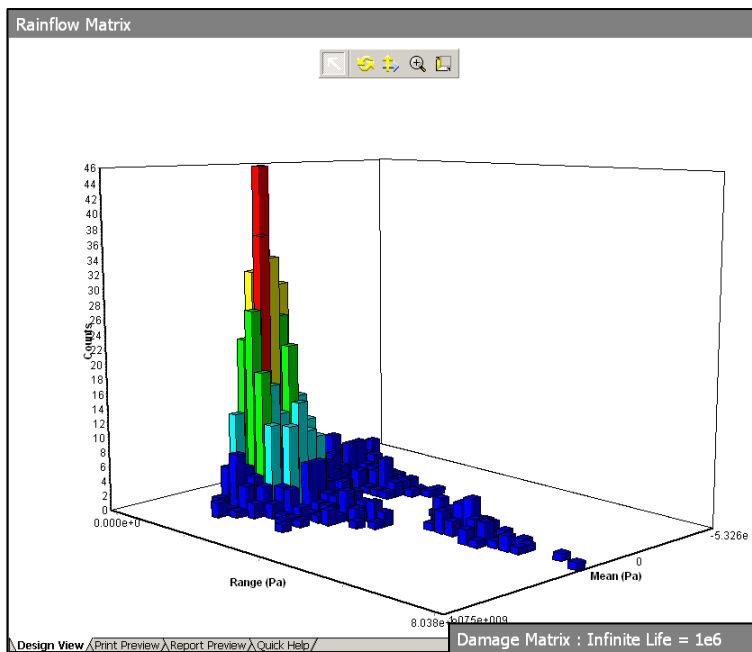
Multiaxial Stress Correction Factors

Experimental test data is mostly uniaxial whereas FE results are usually multiaxial. At some point, stress must be converted from a multiaxial stress state to a uniaxial one. Von-Mises, max shear, maximum principal stress, or any of the component stresses can be used to compare against the experimental uniaxial stress value. A "signed" Von-Mises stress may be chosen where the Von-Mises stress takes the sign of the largest absolute principal stress. This is useful to identify any compressive mean stresses since several of the mean stress theories treat positive and negative mean stresses differently.

Fatigue Modifications

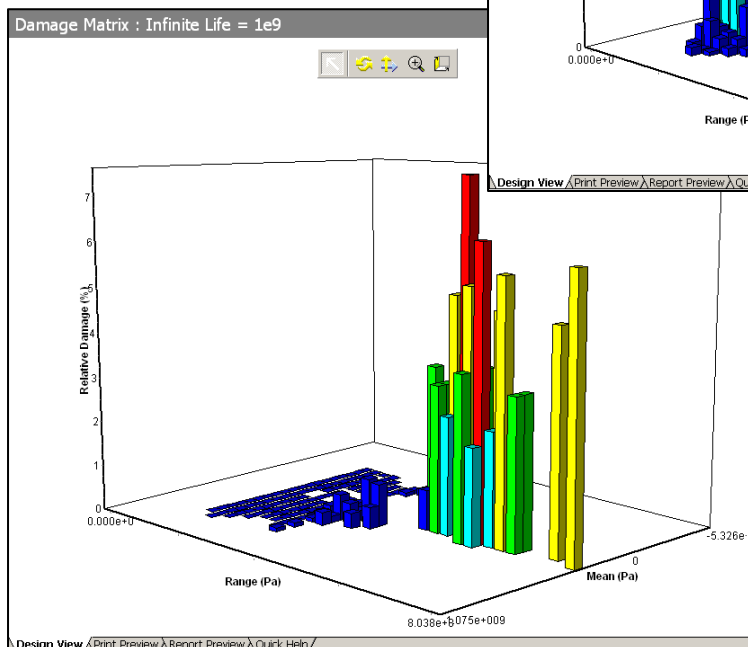
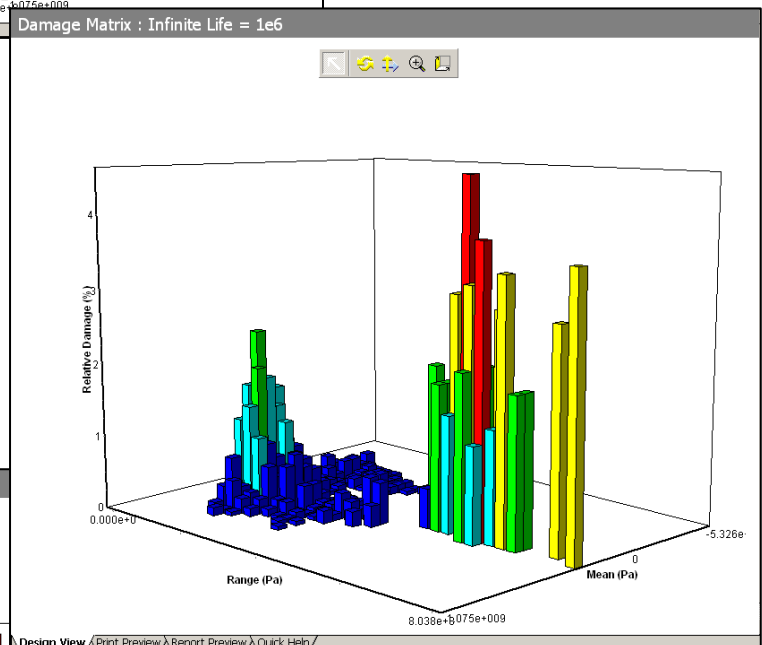
Value of Infinite Life

Another available option when conducting a variable amplitude fatigue analysis is the ability to set the value used for infinite life. In constant amplitude loading, if the alternating stress is lower than the lowest alternating stress on the fatigue curve, the fatigue tool will use the life at the last point. This provides for an added level of safety because many materials do not exhibit an endurance limit. However, in non-constant amplitude loading, cycles with very small alternating stresses may be present and may incorrectly predict too much damage if the number of the small stress cycles is high enough. To help control this, the user can set the infinite life value that will be used if the alternating stress is beyond the limit of the SN curve. Setting a higher value will make small stress cycles less damaging if they occur many times. The rainflow and damage matrix results can be helpful in determining the effects of small stress cycles in your loading history. The rainflow and damage matrices shown in Figure 13 illustrates the possible effects of infinite life. Both damage matrices came from the same loading (and thus same rainflow matrix), but the first damage matrix was calculated with an infinite life of $1e6$ cycles and the second was calculated with an infinite life of $1e9$ cycles.



Rainflow matrix for a given load history.

Here is the resulting Damage Matrix when the Value of Infinite Life is equal to 1e6 cycles. The total damage is calculated to be 0.19.



Here is the resulting Damage Matrix when the Value of Infinite Life is equal to 1e9 cycles. Total damage is calculated to be 0.12 (37% less damage)

Figure 14. Effect of the Value of Infinite Life on fatigue damage.

Fatigue Strength Factor

Fatigue material property tests are usually conducted under very specific and controlled conditions. If the service part conditions differ from the as tested conditions, modification factors can be applied to try to account for the difference. The fatigue alternating stress is usually divided by this modification factor and can be found in design handbooks. (Dividing the alternating stress is equivalent to multiplying the fatigue strength by K_f .) Fatigue Strength Factor (K_f) reduces the fatigue strength and must be less than one. Note that this factor is applied to the alternating stress only and does not affect the mean stress.

Loading Scale Factor

The user may also specify a Loading Scale Factor that will scale all stresses, both alternating and mean by the specified value. This value may be parameterized. Applying a scale factor is useful to avoid having to solve the static model again to see the effects of changing the magnitude of the FEM loads. In addition, this factor may be useful to convert a non-constant amplitude load history data into the appropriate values (See Connecting Rod Under Random Loading).

This concludes the input decisions required to perform either a Stress Life or a Strain Life fatigue analysis. Once the fatigue calculation has been performed, there are a variety of results available that depend on the type of fatigue analysis performed. Some provide contour plots of a specific result while others give supplemental information about the critical location. Let's explore the fatigue results available to us in the next section.

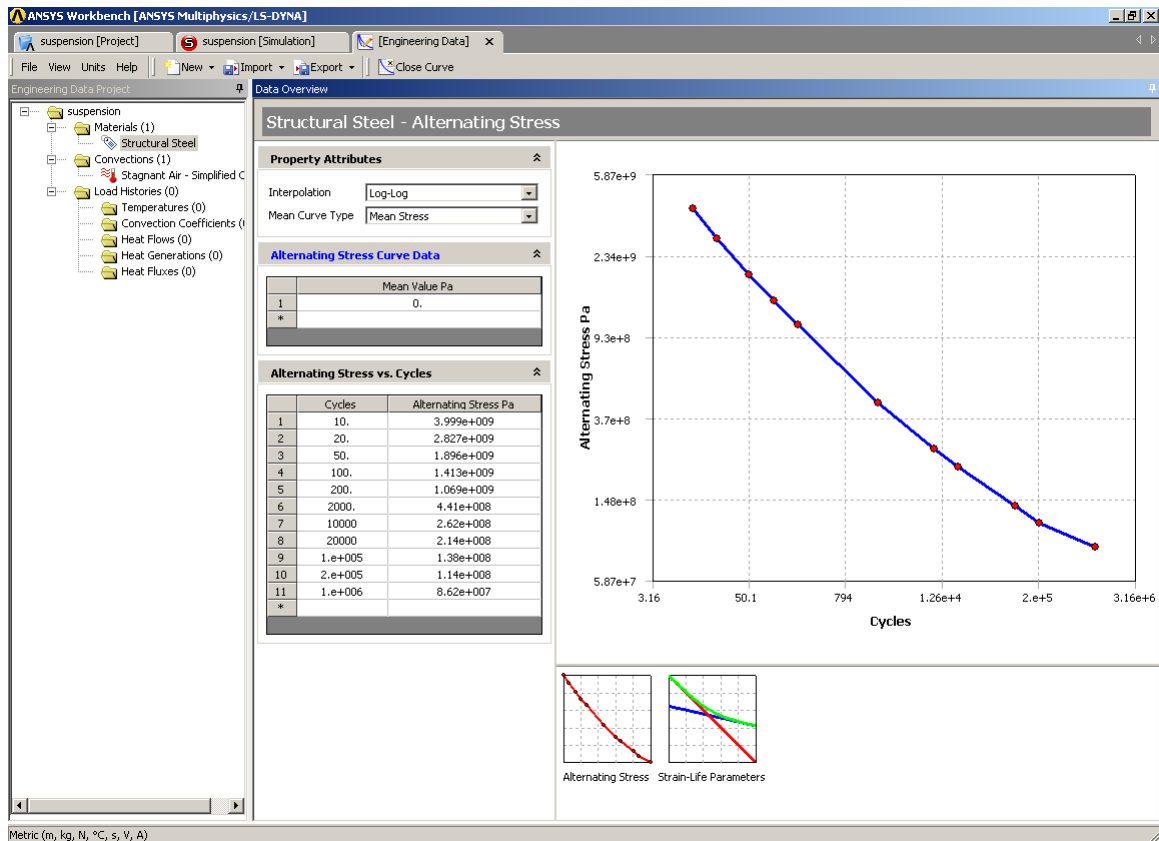


Figure 15. Example of an S-N curve displayed and interpolated as log-log. See Figure 9 for a semi-log display.

Stress Life Interpolation

When the stress life analysis needs to query the S-N curve, almost assuredly the data will not be available at the same stress point as the analysis has produced; hence the stress life analysis needs to interpolate the S-N curve to find an appropriate value. Within a Stress Life analysis, there are three different methods by which interpolations can be done; log-log, semi-log and linear. Results will vary due to the interpolation method used.

Types of Results

Just like some of the input decisions change depending upon whether you are performing a Stress Life or a Strain Life analysis, calculations and results can be dependent upon the type of fatigue analysis. Results can range from contour plots of a specific result over the whole model to information about the most damaged point in the model (or the most damaged point in the scope of the result). Results that are common to both types of fatigue analyses are listed below:

- Fatigue life
- Fatigue damage at a specified design life
- Fatigue factor of safety at a specified design life
- Stress biaxiality
- Fatigue sensitivity chart
- Rainflow matrix output (Beta for Strain Life at 10.0)
- Damage matrix output (Beta for Strain Life at 10.0)

The results that are only available for Stress Life are:

- Equivalent alternating stress

The results that are only available for Strain Life are:

- Hysteresis

General Fatigue Results

Fatigue Life

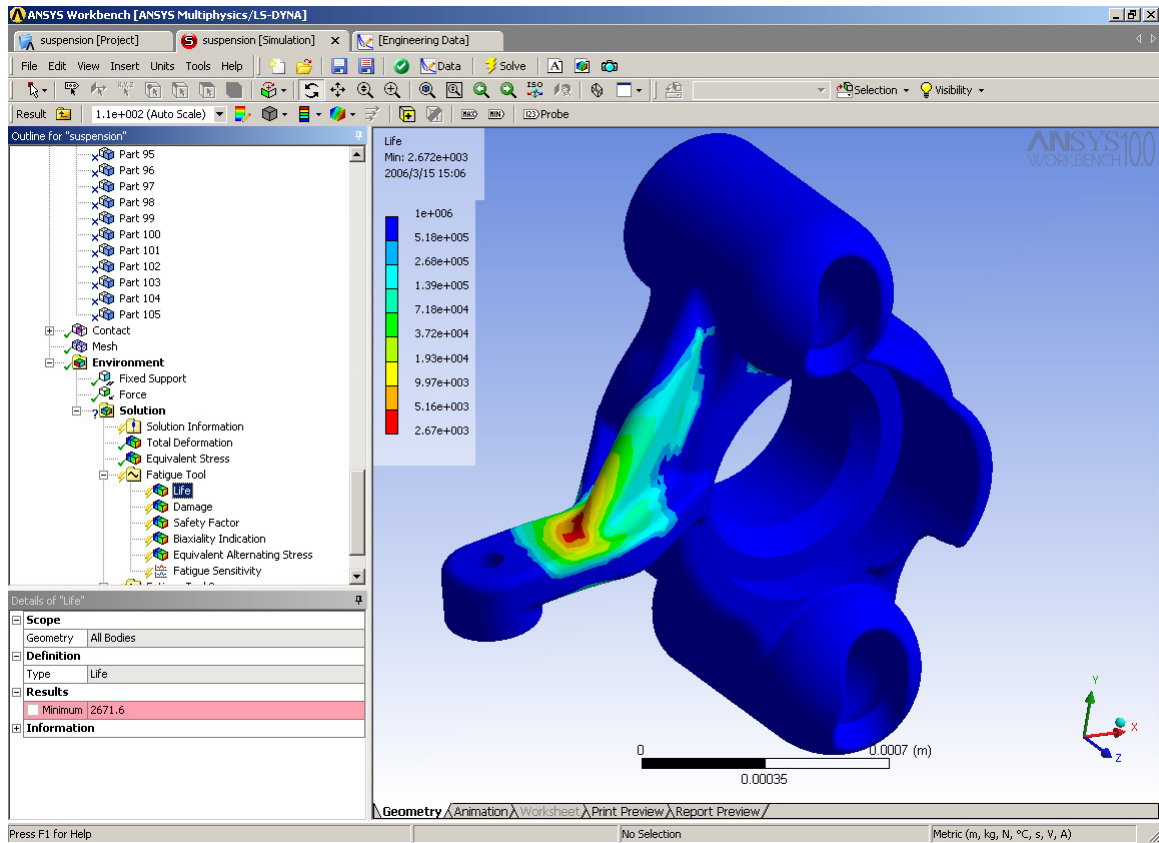


Figure 16. Contour Plot of Fatigue Life over the whole model.

Fatigue Life can be over the whole model or scoped just like any other contour result in Workbench (i.e. parts, surfaces, edges, and vertices). In addition, this and any contour result may be exported to a tab-delimited text file by a right mouse button click on the result. This result contour plot shows the available life for the given fatigue analysis. If loading is of constant amplitude, this represents the number of cycles until the part will fail due to fatigue. If loading is non-constant, this represents the number of loading blocks until failure. Thus if the given load history represents one hour of loading and the life was found to be 24,000, the expected model life would be 1,000 days. In a Stress Life analysis with constant amplitude, if the equivalent alternating stress is lower than the lowest alternating stress defined in the S-N curve, the life at that point will be used.

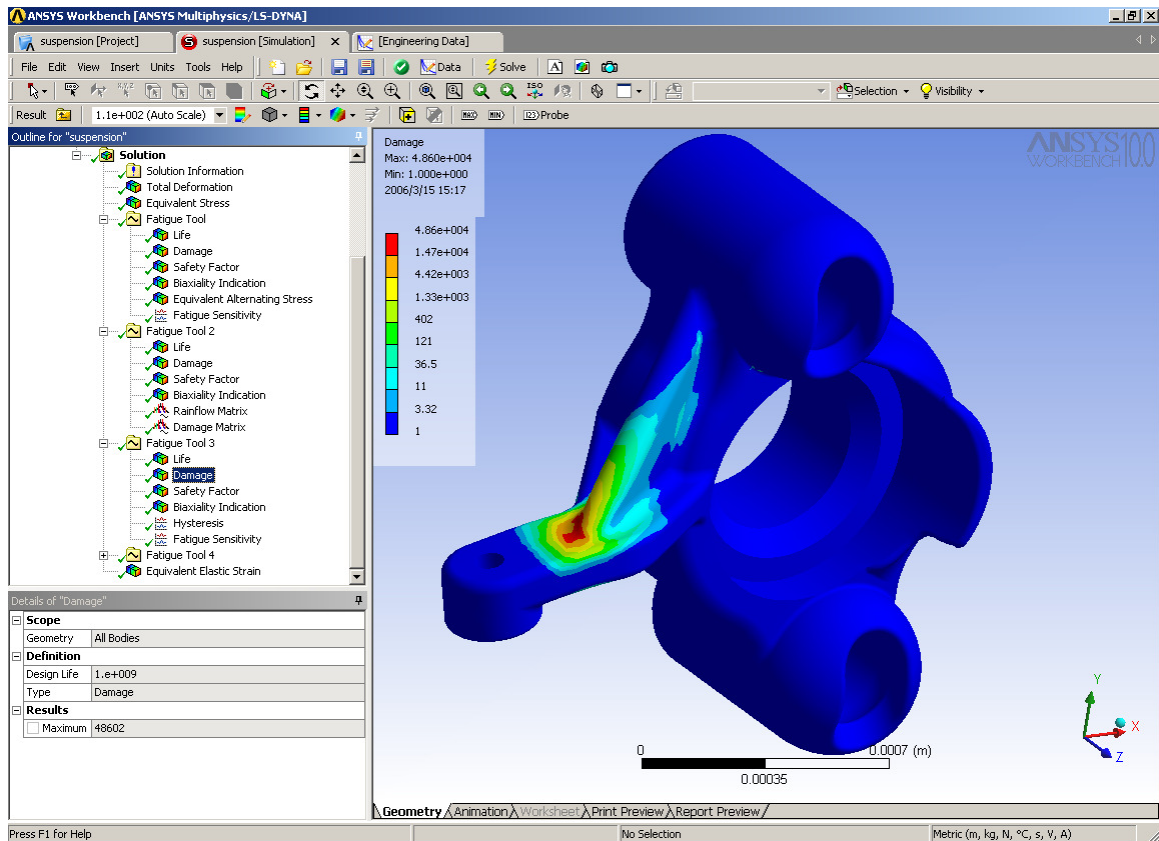


Figure 17. Contour Plot of Fatigue Damage over the whole model.

Fatigue Damage

Fatigue Damage is a contour plot of the fatigue damage at a given design life. Fatigue damage is defined as the design life divided by the available life. This result may be scoped. The default design life may be set through the Control Panel. For Fatigue Damage, values greater than 1 indicate failure before the design life is reached.

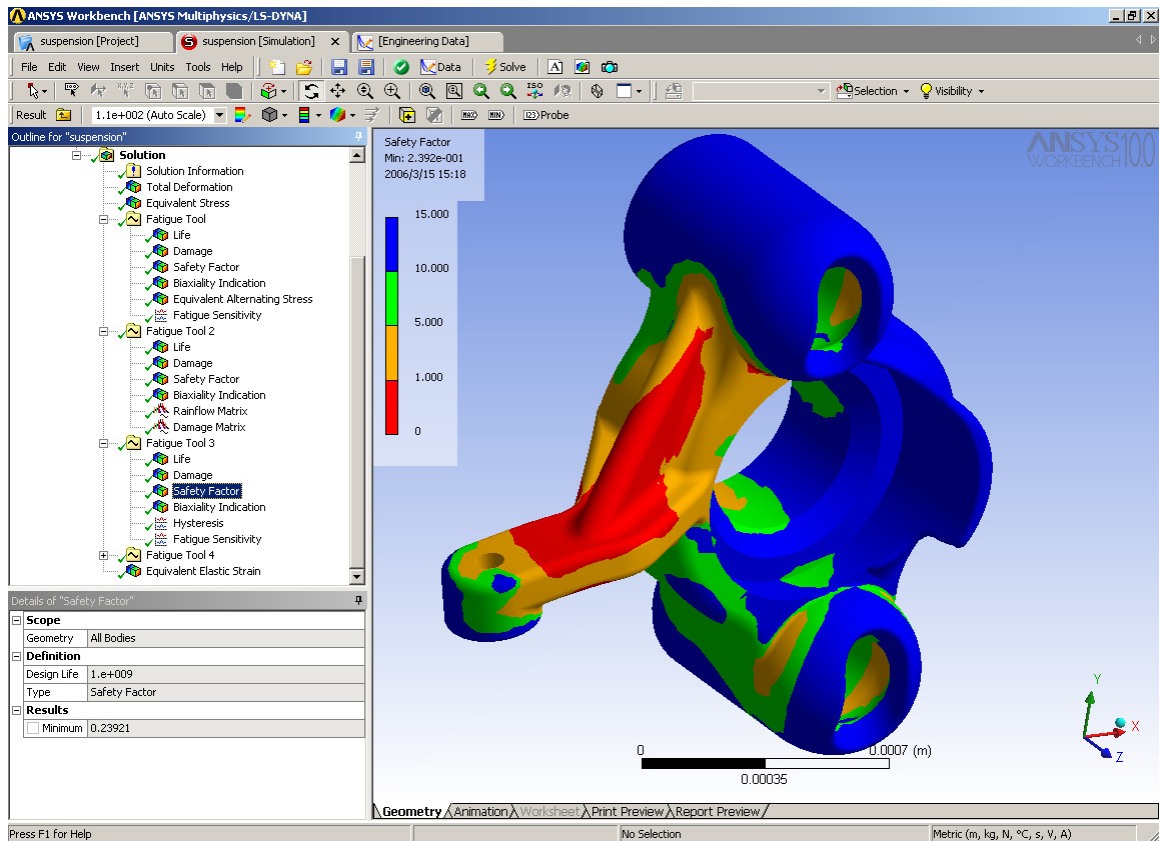


Figure 18. Contour Plot of Fatigue Safety Factor over the whole model.

Fatigue Safety Factor

Fatigue Safety Factor is a contour plot of the factor of safety with respect to a fatigue failure at a given design life. The maximum Factor of Safety displayed is 15. Like damage and life, this result may be scoped. For Fatigue Safety Factor, values less than one indicate failure before the design life is reached.

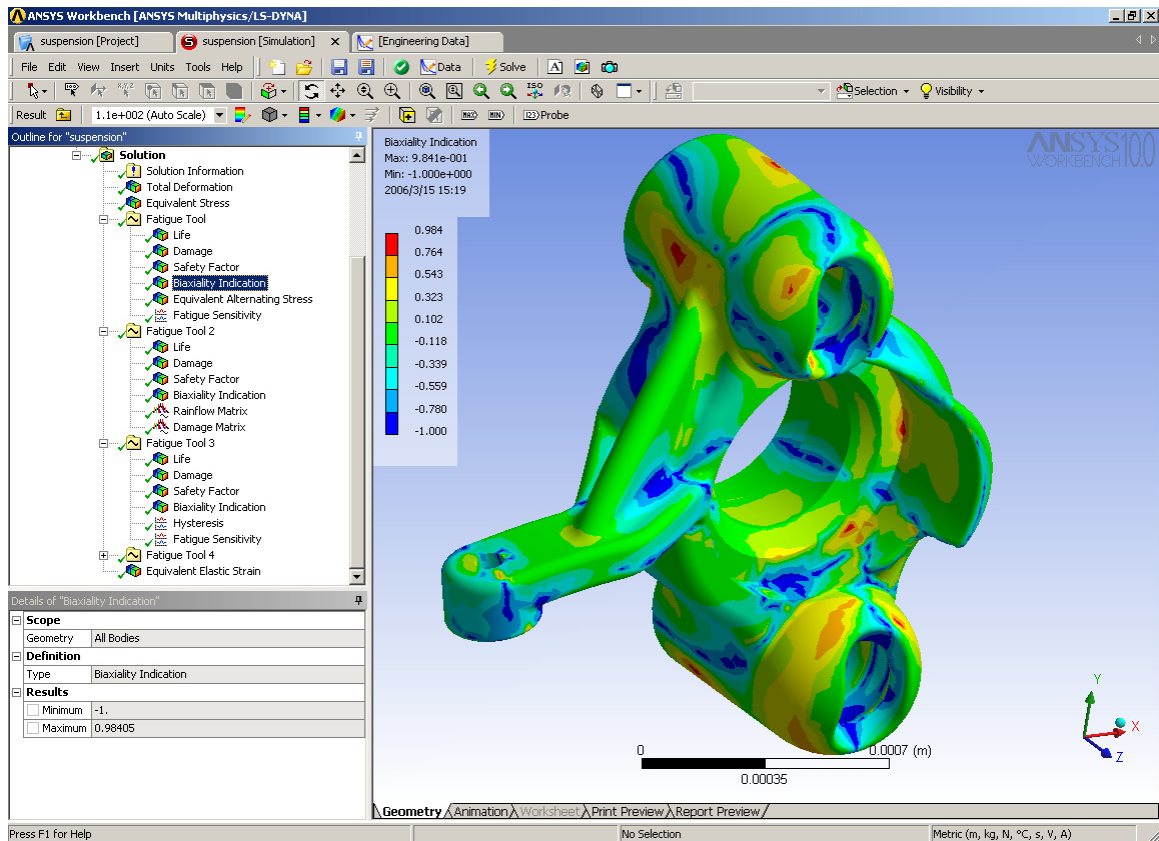


Figure 19. Contour Plot of Biaxiality Indication over the whole model.

Biaxiality Indication

As mentioned previously, fatigue material properties are based on uniaxial stresses but real world stress states are usually multiaxial. This result gives the user some idea of the stress state over the model and how to interpret the results. Biaxiality indication is defined as the principal stress smaller in magnitude divided by the larger principal stress with the principal stress nearest zero ignored. A biaxiality of zero corresponds to uniaxial stress, a value of -1 corresponds to pure shear, and a value of 1 corresponds to a pure biaxial state. As you can see in the Biaxiality Figure, the majority of this model is under a pure uniaxial stress, with parts exhibiting both pure shear and nearly pure biaxiality. When using the biaxiality plot along with the safety factor plot above, it can be seen that the most damaged point occurs at a point of mostly uniaxial stress. If the most damaged spot was under pure shear, then it would be desirable to use S-N data collected through torsional loading if such data was available. Of course collecting experimental data under different loading conditions is cost prohibitive and not often done.

Note that for non-proportional fatigue loading, there are multiple stress states and thus there is no single stress biaxiality at each node. Thus if the fatigue tool has non-proportional loading, the user may select either to view the average or standard deviation of stress biaxiality. The average value may be interpreted as above and in combination with the standard deviation, the user can get a measure of how the stress state changes at a given location. Thus a small standard deviation indicates a condition where the loading is near proportional while a larger deviation indicates change in the direction of the principal stress vectors. This information can be used to give the user additional confidence in his results or whether more in depth fatigue analysis is needed to account for non-proportionality.

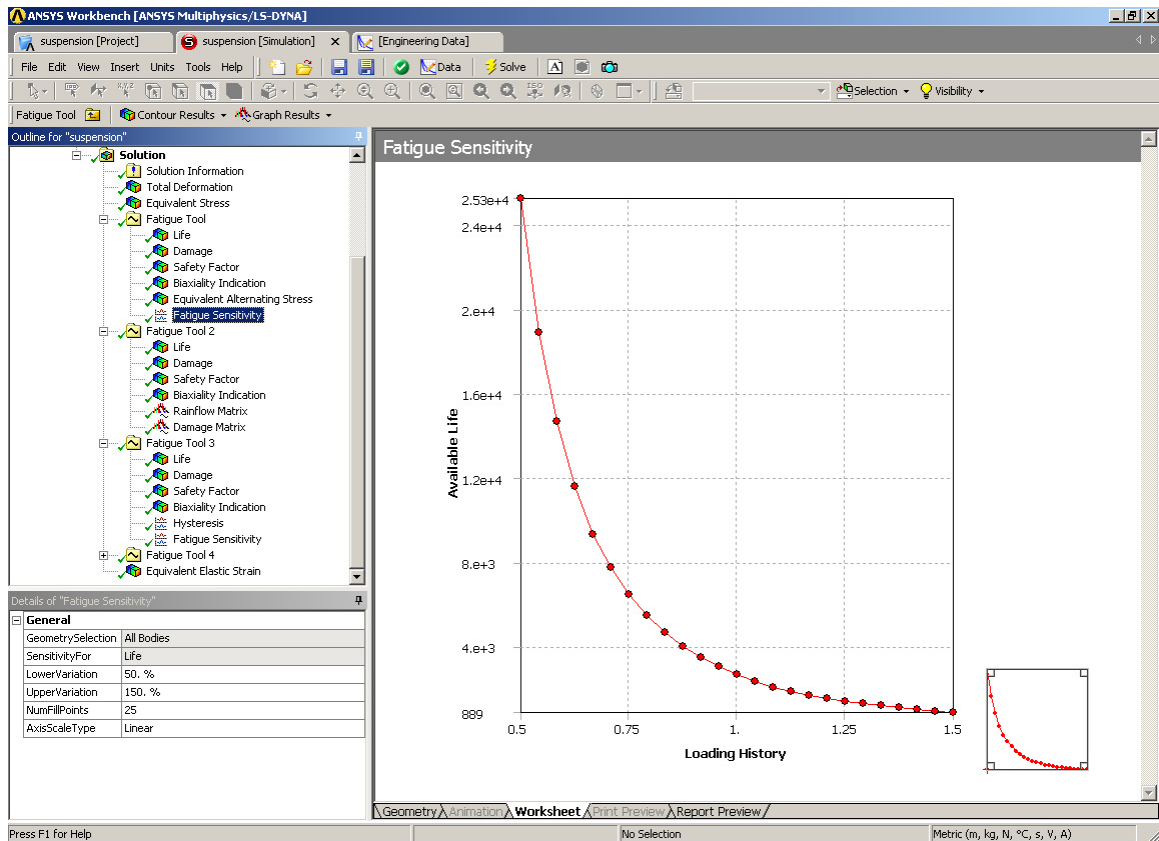


Figure 20. Example of a Fatigue Sensitivity curve.

Fatigue Sensitivity

Fatigue Sensitivity shows how the fatigue results change as a function of the loading at the critical location on the model. This result may be scoped. Sensitivity may be found for life, damage, or factor of safety. The user may set the number of fill points as well as the load variation limits. For example, the user may wish to see the sensitivity of the model's life if the FE load was 50% of the current load up to if the load 150% of the current load. A value of 100% corresponds to the life at the current loading on the model. Negative variations are allowed in order to see the effects of a possible negative mean stress if the loading is not totally reversed. Linear, Log-X, Log-Y, or Log-Log scaling can be chosen for chart display. Default values for the sensitivity options may be set through the Control Panel.

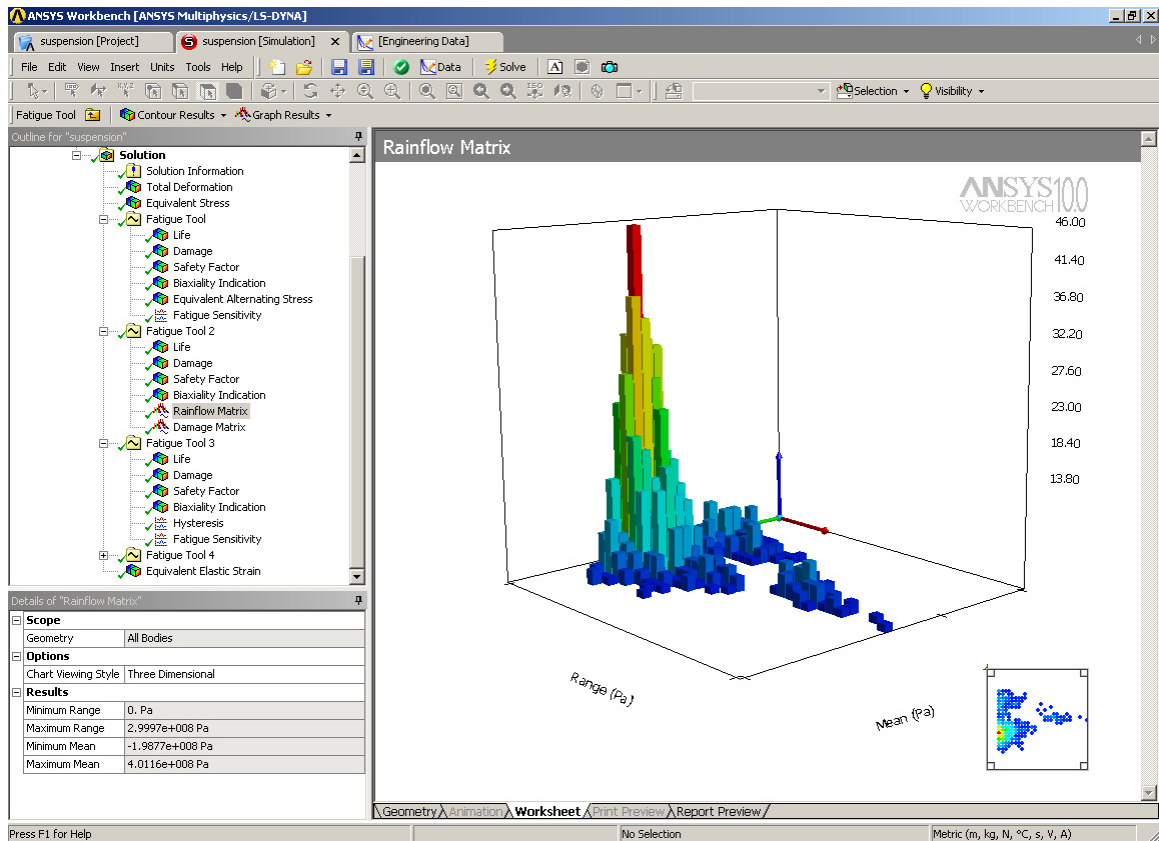


Figure 21. Rainflow Matrix Chart showing percent of occurrence.

Rainflow Matrix Chart (Beta for Strain Life at 10.0)

Rainflow Matrix Chart is a plot of the rainflow matrix at the critical location. This result is only applicable for non-constant amplitude loading where rainflow counting is needed. This result may be scoped. In this 3-D histogram, alternating and mean stress is divided into bins and plotted. The Z-axis corresponds to the number of counts for a given alternating and mean stress bin. This result gives the user a measure of the composition of a loading history. (Such as if most of the alternating stress cycles occur at a negative mean stress.) From the rainflow matrix figure, the user can see that most of the alternating stresses have a positive mean stress and that in this case the majority of alternating stresses are quite low.

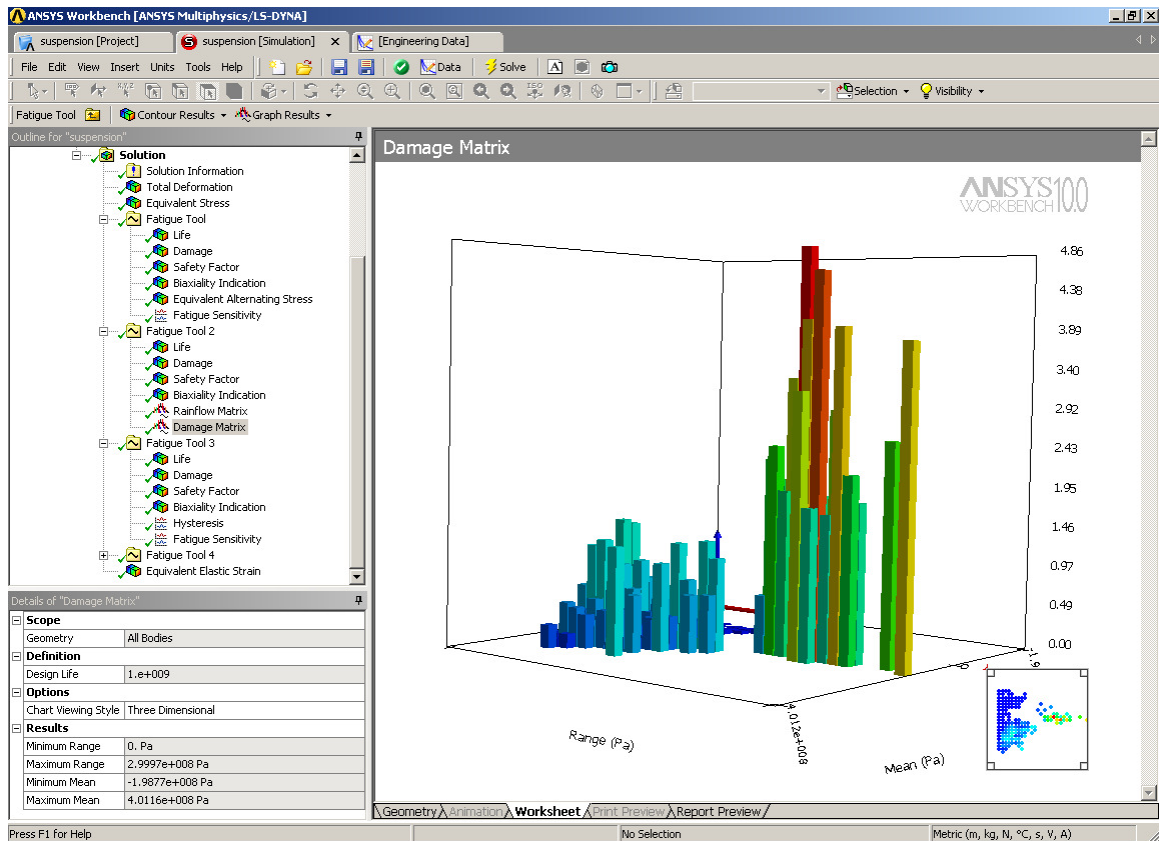


Figure 22. Damage Matrix Chart showing percent damage.

Damage Matrix Chart (Beta for Strain Life at 10.0)

Damage Matrix Chart is a plot of the damage matrix at the critical location on the model. This result is only applicable for non-constant amplitude loading where rainflow counting is needed. This result may be scoped. This result is similar to the rainflow matrix except that the percent damage that each of the Rainflow bin cause is plotted as the Z-axis. As can be seen from the corresponding damage matrix for the above rainflow matrix, in this particular case although most of the counts occur at the lower stress amplitudes, most of the damage occurs at the higher stress amplitudes.

Fatigue Results Unique to Stress Life Fatigue Analysis

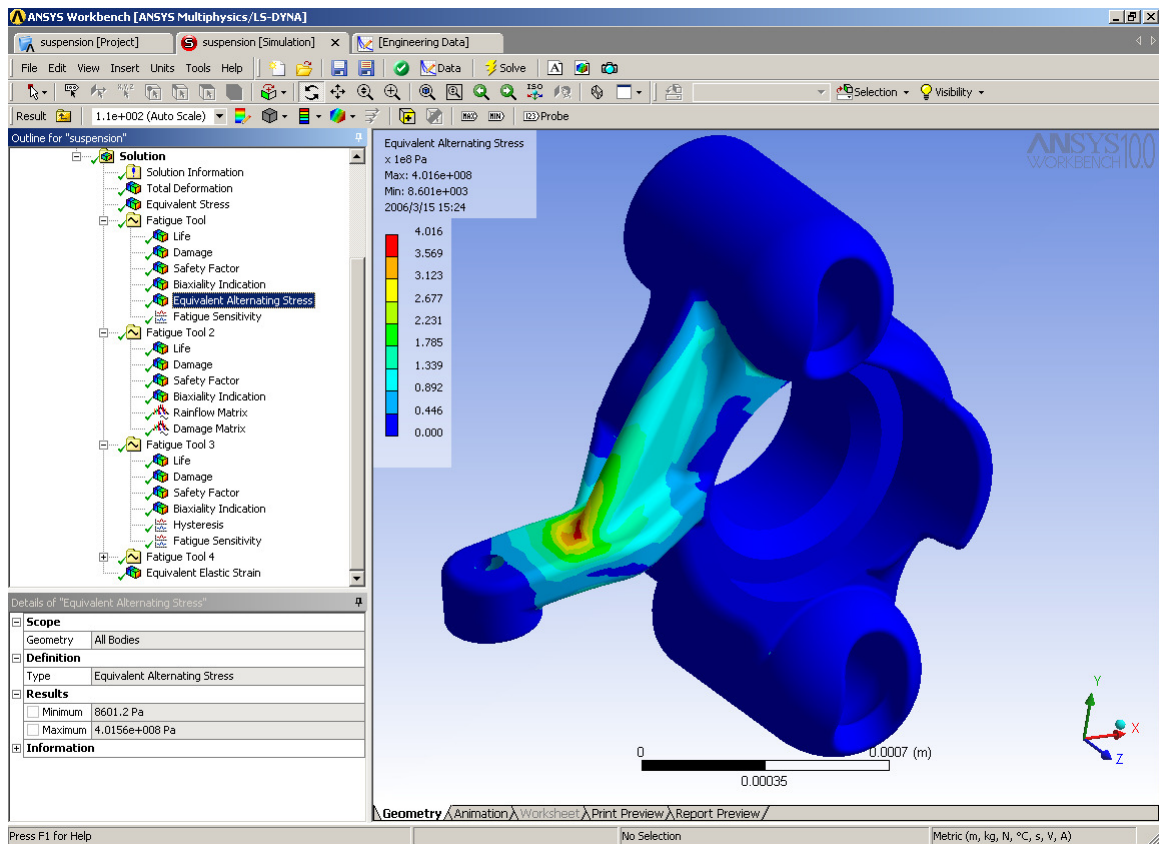


Figure 23. Contour Plot of Equivalent Alternating Stress.

Equivalent Alternating Stress

In a Stress Life fatigue analysis, one always needs to query an SN curve to relate the fatigue life to the stress state. Thus the "equivalent alternating stress" is the stress used to query the fatigue SN curve after accounting for fatigue loading type, mean stress effects, multiaxial effects, and any other factors in the fatigue analysis. Thus in a fatigue analysis, the equivalent alternating stress can be thought of as the last calculated quantity before determining the fatigue life. The usefulness of this result is that in general it contains all of the fatigue related calculations independent of any fatigue material properties. As discussed in Part 1, some mean stress theories use static material properties such as tensile strength so Equivalent Alternating Stress may not be totally devoid of material properties. A quantity such as Equivalent Alternating Stress may be useful in a variety of situations:

- Instead of possible security issues with proprietary material stress life properties, an engineer may be given an "equivalent alternating stress" design criteria.
- The equivalent alternating stress may be exported to a 3rd party or "in house" fatigue code that performs specialized fatigue calculations based on the industry specific knowledge.
- An engineer can perform a comparative analysis among a variety of designs using a result type (stress) that he may feel more comfortable with.

- A part can be geometrically optimized with respect to fatigue without regard to the specific material or finishing operations that are going to be used for the final product.

This result is not applicable to Strain Life or Stress life with non-constant amplitude fatigue loading due to the fact multiple SN queries per location are required and thus no single equivalent alternating stress exists.

Fatigue Results Unique to Strain Life Fatigue Analysis

Hysteresis

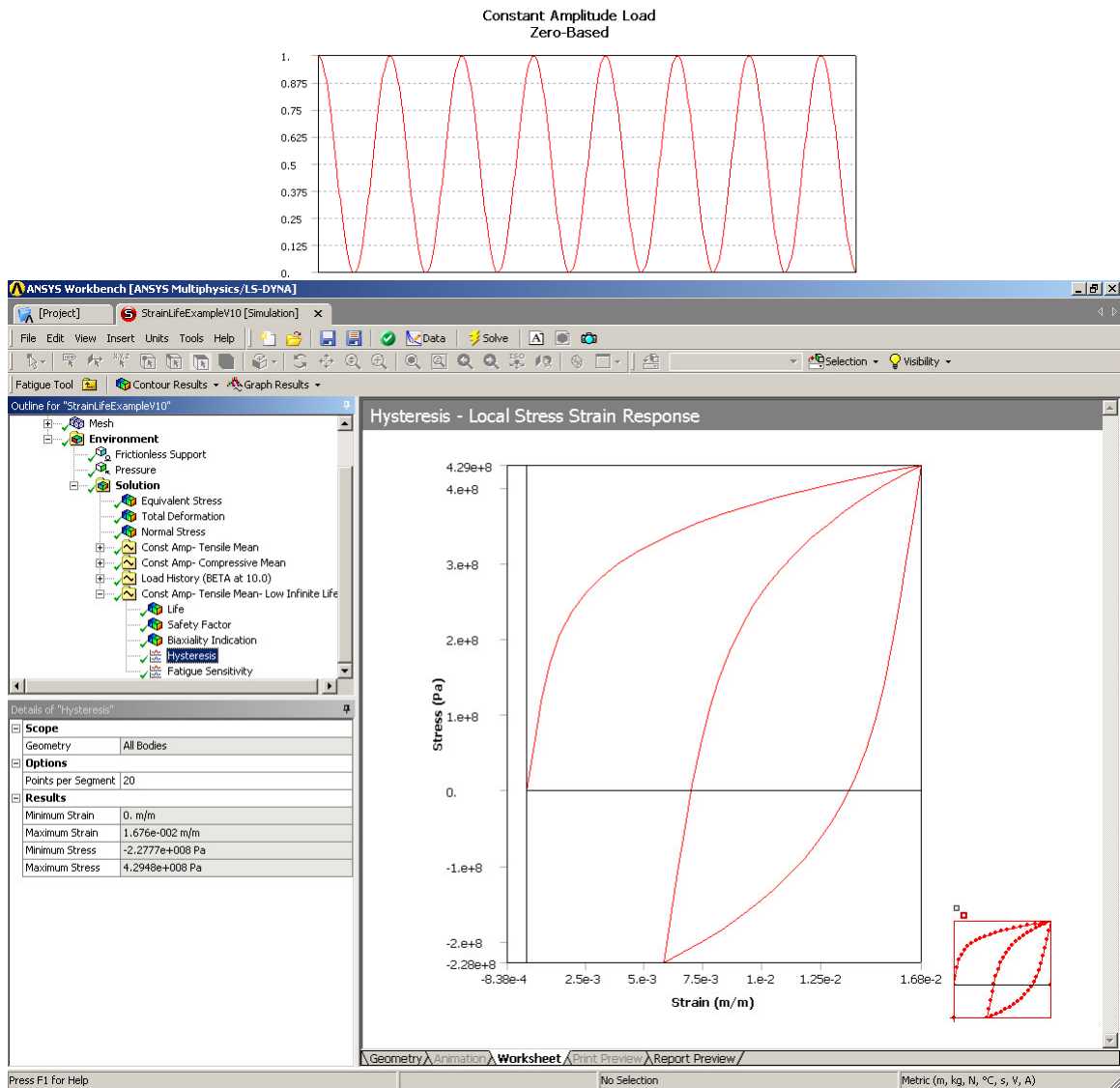


Figure 24. Example of a Hysteresis diagram for a zero based load case (Shown above Hysteresis curve).

In a strain-life fatigue analysis, although the finite element response may be linear, the local elastic/plastic response may not be linear. The Neuber correction is used to determine the local

elastic/plastic response given a linear elastic input. Repeated loading will form closed hysteresis loops as a result of this nonlinear local response. In a constant amplitude analysis a single hysteresis loop is created although numerous loops may be created via rainflow counting in a non-constant amplitude analysis. The Hysteresis result plots the local elastic-plastic response at the critical location of the scoped result (the Hysteresis result can be scoped, similar to all result items). Hysteresis is a good result to help you understand the true local response that may not be easy to infer. Notice in the example above, that although the loading/elastic result is tensile, the local response does venture into the compressive region due to residual stresses created by the plastic response.

Conclusion

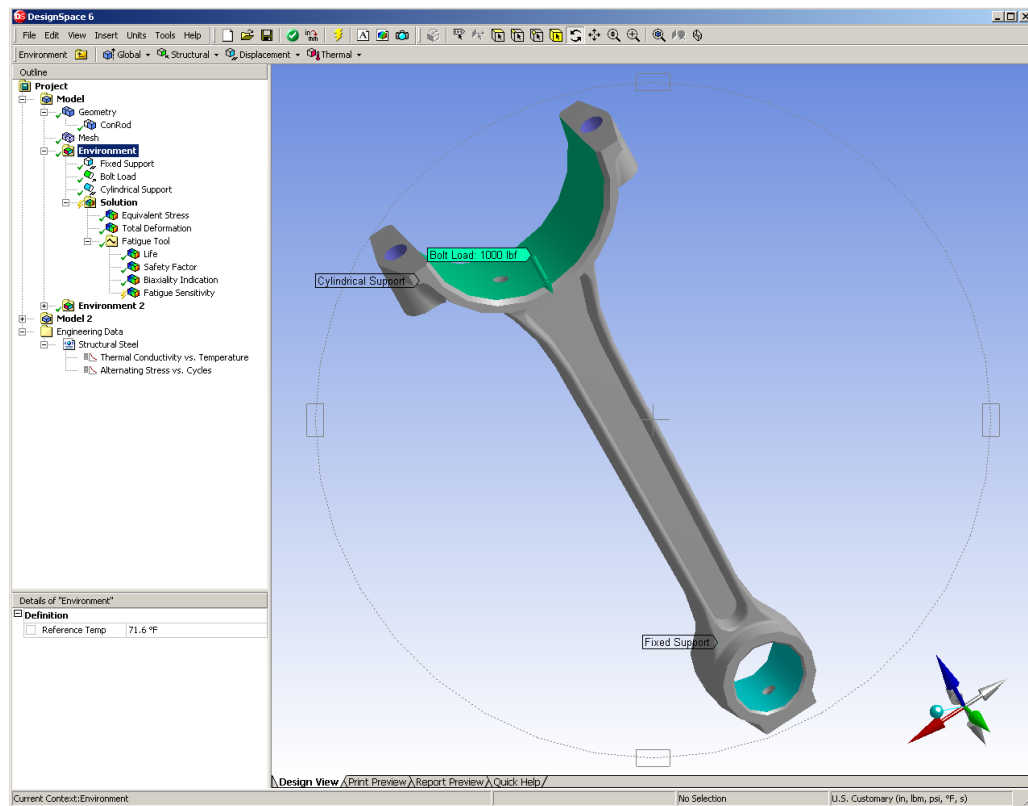
The ANSYS Fatigue Module supports a wide variety of fatigue analysis. The comprehensive calculations and results enable engineers to evaluate their designs for avoiding failures under real world conditions. As a module that is integrated into the ANSYS Workbench Environment, the ANSYS Fatigue Module can further leverage advances in CAD support including Bi-Directional Parameters, Solid Modeling, Virtual Topology, Robust Meshing, Hex-Dominant Meshing, Automatic Contact Detection, Optimization, Design for Six Sigma and Robust Design that the ANSYS Workbench offers.

Typical Use Cases

Connecting Rod Under Fully Reversed Loading

(Model: fatigueUseCaseExample.dsdb) Here we have a connecting rod in a compressor under fully reversed loading (load is applied, removed, then applied in the opposite direction with a max loading of 1000 pounds).

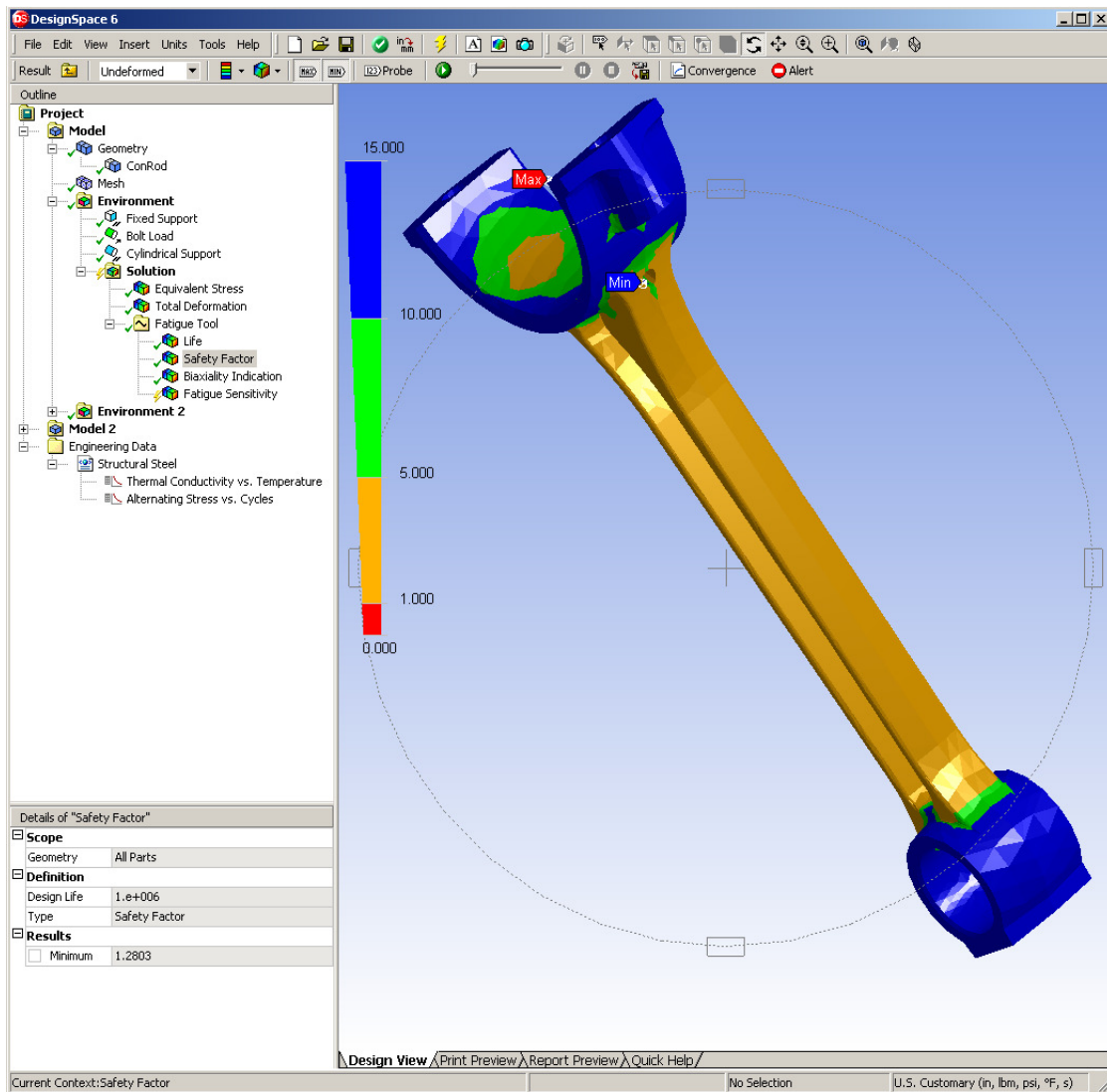
1. Import geometry and apply boundary conditions. Apply loading corresponding to the maximum developed load of 1000 pounds.



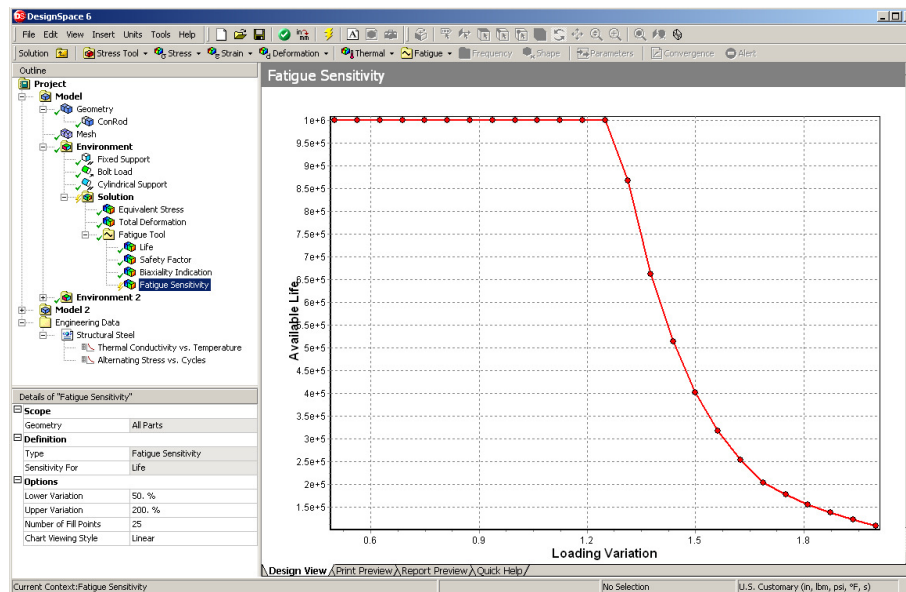
2. Insert fatigue tool.
3. Specify fully reversed loading to create alternating stress cycles.
4. Specify that this is a stress-life fatigue analysis. No mean stress theory needs to be specified since no mean stress will exist (fully reversed loading). Specify that Von-Mises

Details of "Fatigue Tool"	
<input checked="" type="checkbox"/> Materials	
Fatigue Strength Factor (Kf)	0.8
<input checked="" type="checkbox"/> Loading	
Type	Fully Reversed
<input type="checkbox"/> Scale Factor	1.
<input checked="" type="checkbox"/> Options	
Analysis Type	SN-None
Stress Component	Equivalent (Von Mises)

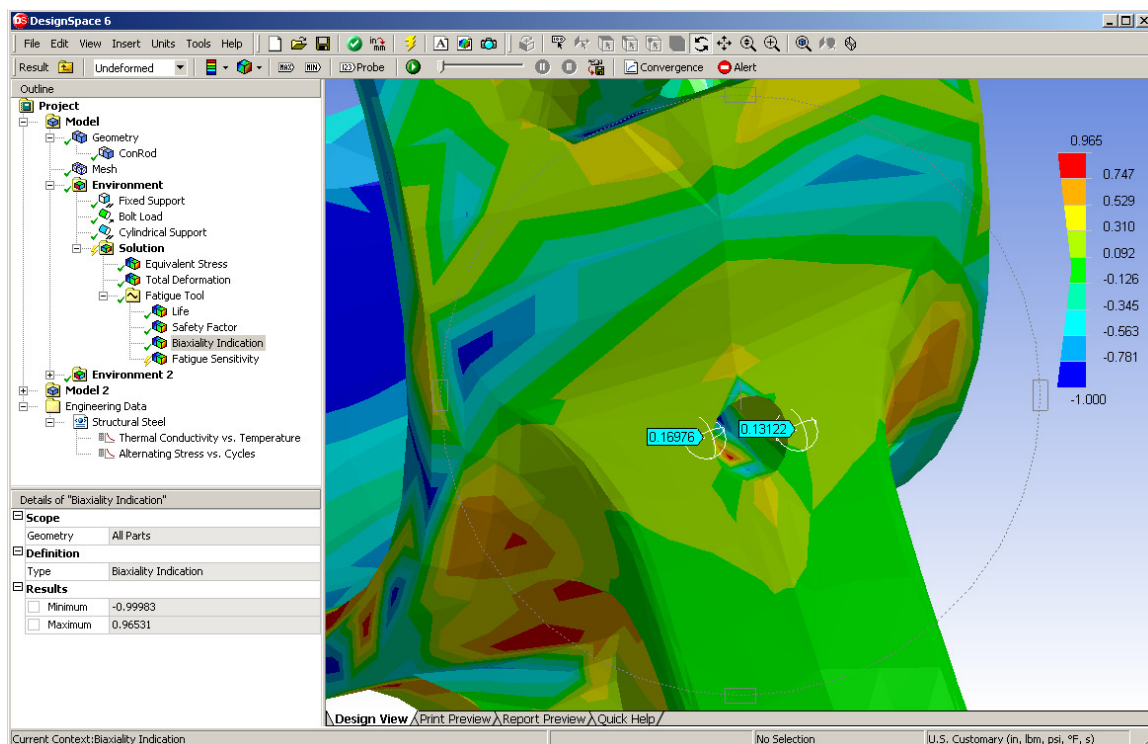
- stress will be used to compare against fatigue material data.
5. Specify a modification factor of .8 since material data represents a polished specimen and the in-service component is cast.
 6. Perform stress and fatigue calculations (Solve command in context menu).
 7. Plot factor of safety for a design life of 1,000,000 cycles.



8. Find the sensitivity of available life with respect to loading. Specify a minimum base load variation of 50% (an alternating stress of 500 lbs.) and a maximum base load variation of 200% (an alternating stress of 2000 lbs.)



9. Determine multiaxial stress state (uniaxial, shear, biaxial, or mixed) at critical life location by inserting "biaxiality indicator" into fatigue tool. The stress state near the critical location is not far from uniaxial (.1~.2), which gives an added measure of confidence since the material properties are uniaxial.



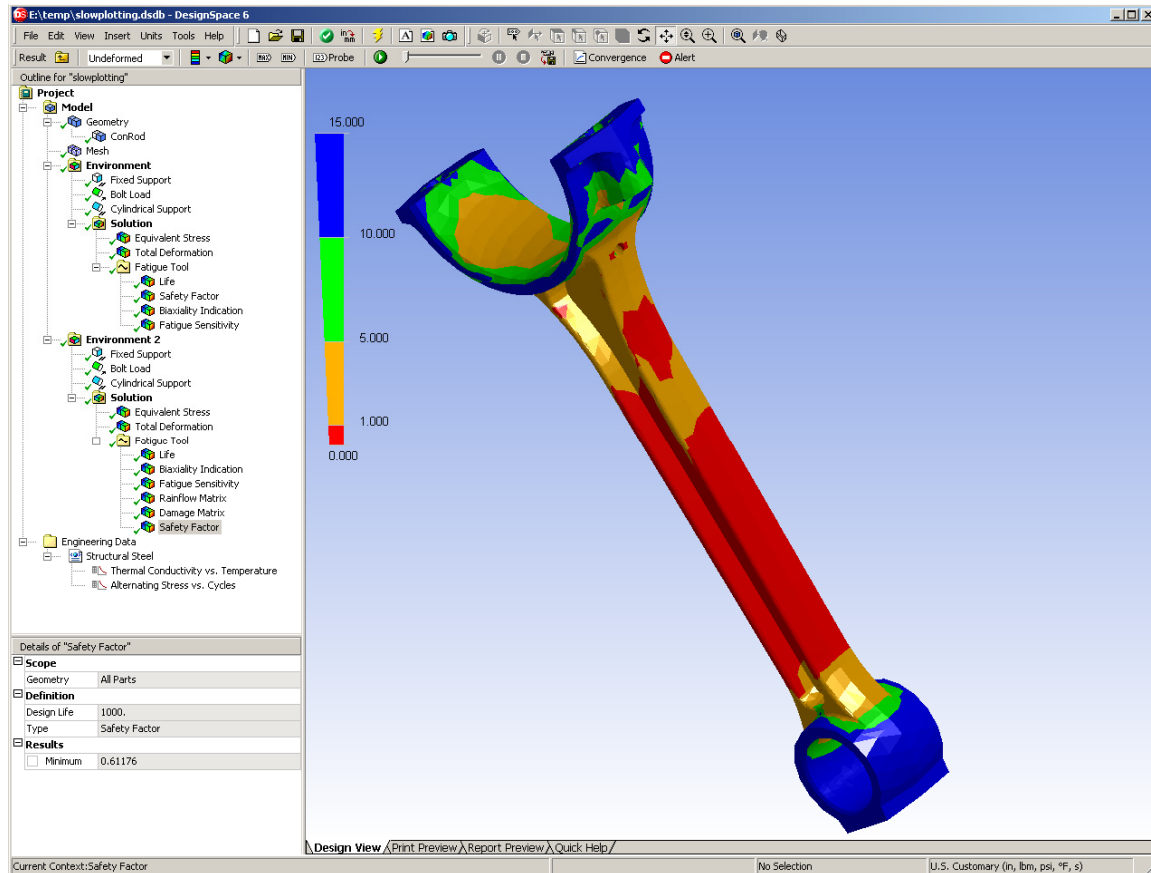
Connecting Rod Under Random Loading

(Model: fatigueUseCaseExample.dsdb) Here we have the same connecting rod and boundary conditions but the loading is not of a constant amplitude over time. Assume that we have strain gauge results that were collected experimentally from the component and that we know that a strain gauge reading of 200 corresponds to an applied load of 1,000 pounds.

1. Conduct the static stress analysis as before using a load of 1,000 pounds.
2. Insert fatigue tool.
3. Specify fatigue loading as coming from a scale history and select scale history file containing strain gauge results over time (ex. Common Files\Ansys Inc\Engineering Data\Load Histories\SAEBracketHistory.dat). Define the scale factor to be .005. (We must normalize the load history so that the FEM load matches the scale factors in the load history file).

$$\left(\frac{1 \text{ FEM load}}{1000 \text{ lbs}} \right) \times \left(\frac{1000 \text{ lbs}}{200 \text{ strain gauge}} \right) = \left(\frac{1 \text{ FEM load}}{200 \text{ strain gauge}} \right) = \text{needed load scale factor}$$

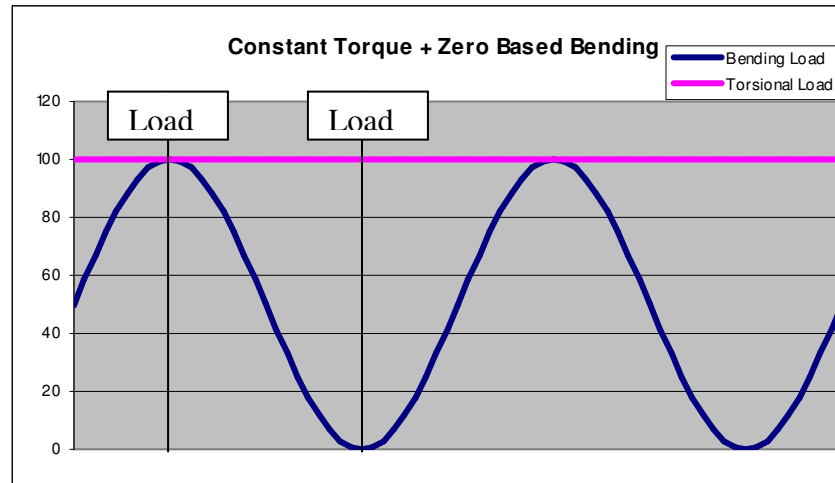
4. Specify a bin size of 32 (Rainflow and damage matrices will be of dimension 32x32).
5. Specify Goodman theory to account for mean-stress effects. (The chosen theory will be illustrated graphically in the graphics window. Specify that a signed Von-Mises stress will be used to compare against fatigue material data. (Use signed since Goodman theory treats negative and positive mean stresses differently.)
6. Perform fatigue calculations (Solve command in context menu).
7. View rainflow and damage matrix.



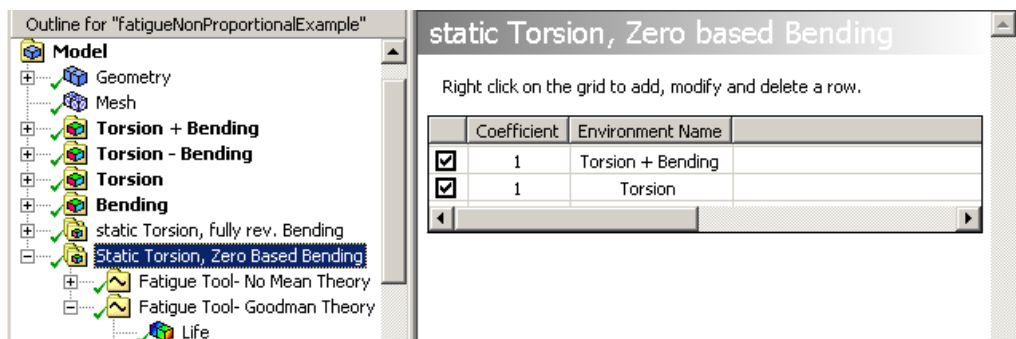
8. Plot life, damage, and factor of safety contours over the model at a design life of 1000. (The fatigue damage and FS if this loading history was experienced 1000 times). Thus if the loading history corresponded to the loading experienced by the part over a month's time, the damage and FS will be at a design life of 1000 months. Note that although a life of only 88 loading blocks is calculated, the needed scale factor (since $FS@1000 = .61$) is only .61 to reach a life of 1000 blocks.
9. Plot factor of safety as a function of the base load (fatigue sensitivity plot, a 2-D XY plot).
10. Copy and paste to create another fatigue tool and specify that mean stress effects will be ignored (SN-None theory) This will be done to ascertain to what extent mean stress is affecting fatigue life.
11. Perform fatigue calculations.
12. View damage and factor of safety and compare results obtained when using Goodman theory to get the extent of any possible mean stress effect.
13. Change bin size to 50, rerun analysis, and compare fatigue results to verify that the bin size of 32 was of adequate size to get desired precision for alternating and mean stress bins.

Universal Joint Under Non-Proportional Loading

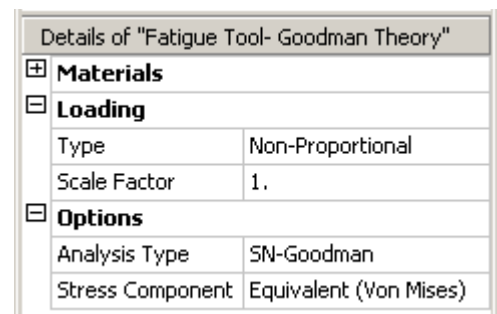
(Model: fatigueNonProportionalExample.dsdb) In this case we want to simulate a fatigue loading on a u-joint with a constant torque load that has a zero based bending load. The applied torque is a constant 1000 lb-ft and the bending force alternates between 0 and 1000 lb. Thus 2 loadings are required to model this scenario. Load 1 will contain the torque plus the bending load. For load 2, the bending force is zero so only the torque is required.



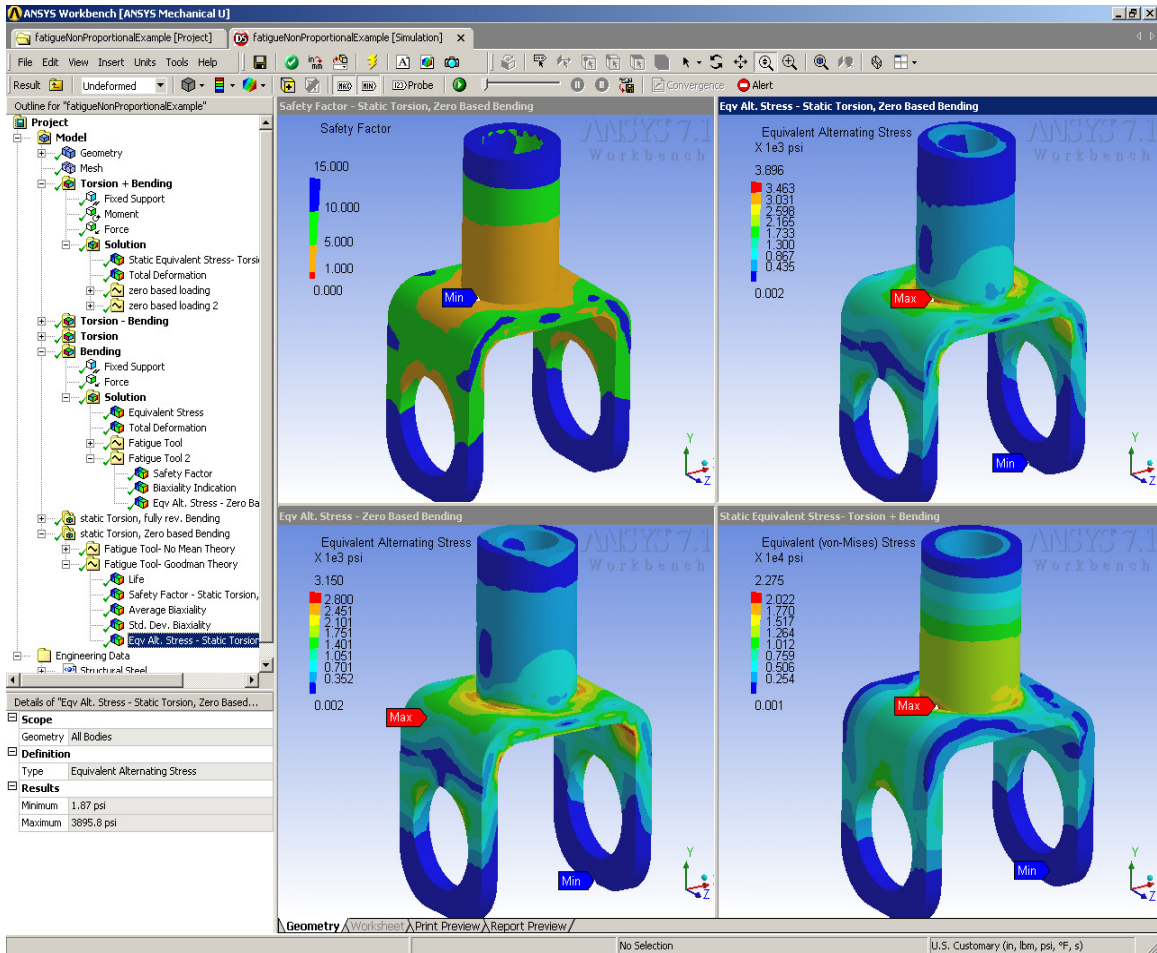
1. Create an environment called "torsion+bending" that contains both the force and the moment. Insert total deformation and SEQV stress.
2. Duplicate this environment, rename it to "torsion" and delete the moment load.
3. Insert a solution combination folder and add the 2 environments to the list.



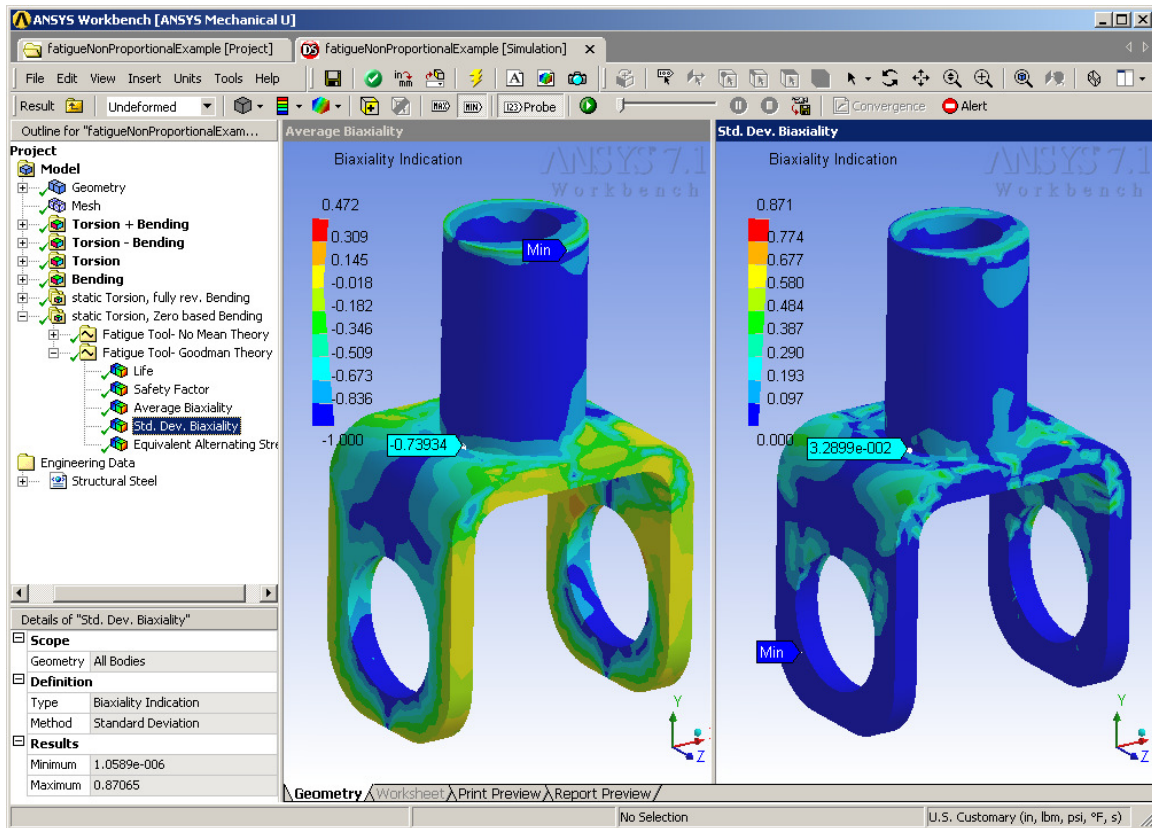
4. Insert a fatigue tool under the solution combination folder.
5. Select the loading type to "Non-proportional". This is important else the fatigue tool will operate on superposition of the load cases instead of alternating between them.
6. Select "SN-Goodman" for the mean stress correction theory.
7. Perform stress and fatigue calculations (Solve command in context menu).
8. Plot factor of safety for a design life of 1e6 cycles.



9. Plot equivalent alternating stress and compare to equivalent alternating stress from a pure bending fatigue load as well as equivalent stress from static bending+torsional load. Note that the equivalent alternating stress patterns from the non-proportional loading fatigue calculations differ from the others (as expected).



10. Plot average and standard deviation of stress biaxiality. Note that at the critical location, the average stress biaxiality is near -1 (pure shear) and the standard deviation is small.



11. Analyze different loading scenarios such static torsion + fully reversed bending (Env1=Torsion+Bending, Env2=Torsion-Bending) or zero based torsion and bending fully out of phase (Env1=Torsion, Env2=Bending) by creating the various solution combinations as desired.

Fatigue References

1. Hancq, D.A., Walters, A.J., Beuth, J.L., "Development of an Object-Oriented Fatigue Tool", *Engineering with Computers*, Vol 16, 2000, pp. 131-144.
This paper gives details on both the underlying structure and engineering aspects of the fatigue tool.
2. Bannantine, J., Comer, J., Handrock, J. "Fundamentals of Metal Fatigue Analysis", New Jersey, Prentice Hall (1990).
This is an excellent book that explains the fundamentals of fatigue to a novice user. Many topics such as mean stress effects and rainflow counting are topics in this book.
3. Stephens, Ralph I., Fatemi, Ali, Stephens, Robert R., Fuchs, Henry O. "Metal Fatigue in Engineering", New York, John Wiley and Sons, Inc. (2001)
4. Lampman, S.R. editor, "ASM Handbook: Volume 19, Fatigue and Fracture", ASM International (1996).
Good reference to have when conducting a fatigue analysis. Contains papers on a wide variety of fatigue topics.
5. U.S. Dept. of Defense, "MIL-HDBK-5H: "Metallic materials and Elements for Aerospace Vehicle Structures", (1998).
This publication distributed by the United States government gives fatigue material properties of several common engineering alloys. It is freely downloadable over the Internet from the NASA website. <http://analyst.gsfc.nasa.gov/FEMCI/links.html>

Revision History

- Original Document Created, Release 10.0 SP1, March 20, 2006