Analyzing **Random Vibration Fatigue**

Powerful ANSYS Workbench tools help calculate the damage of vibrations that lack straightforward cyclic repetition.

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Determining the fatigue life of parts under periodic, sinusoidal vibration is a fairly straightforward process in which damage content is calculated by multiplying the stress amplitude of each cycle from harmonic analysis with the number of cycles that the parts experience in the field. The computation is relatively simple because the absolute value of the vibration is highly predictable at any point in time.

Vibrations may be random in nature in a wide range of applications, however, such as vehicles traveling on rough roads or industrial equipment operating in the field where arbitrary loads may be encountered. In these cases, instantaneous vibration amplitudes are not highly predictable as the amplitude at any point in time is not related to that at any other point in time. As shown in Figure 1, the lack of periodicity is apparent with random vibrations.

The complex nature of random vibrations is demonstrated with a Fourier analysis of the random time–history shown in Figure 2, revealing that the random motion can be represented as a series of many overlapping sine waves, with each curve cycling at its own frequency and amplitude. With these multiple frequencies occurring at the same time, the structural resonances of different components can be excited simultaneously, thus increasing the potential damage of random vibrations.

Statistical Measures of Random Vibration

Because of the mathematical complexity of working with these overlapping sine curves to find instantaneous amplitude as an exact function of time, a more efficient way of dealing with random vibrations is to use a statistical process to determine the probability of the occurrence of particular amplitudes. In this type of approach, the random vibration can be characterized using a mean, the standard deviation and a probability distribution. Individual vibration amplitudes are not determined. Rather, the amplitudes are averaged over a large number of cycles and the cumulative effect determined for this time period. This provides a more practical process for characterizing random vibrations than analyzing an unimaginably large set of time–history data for many different vibration profiles.

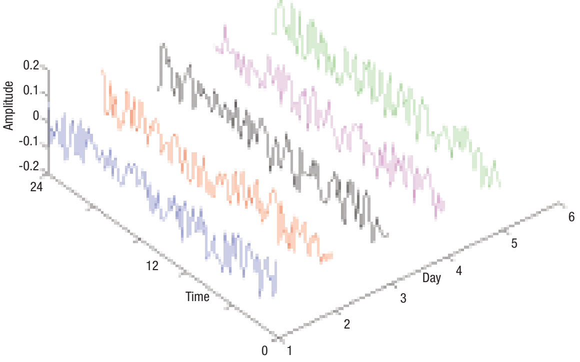

Figure 1. Random vibrations measured for vehicle on a rough road showing periodicity for single, dual and quad disk configuration

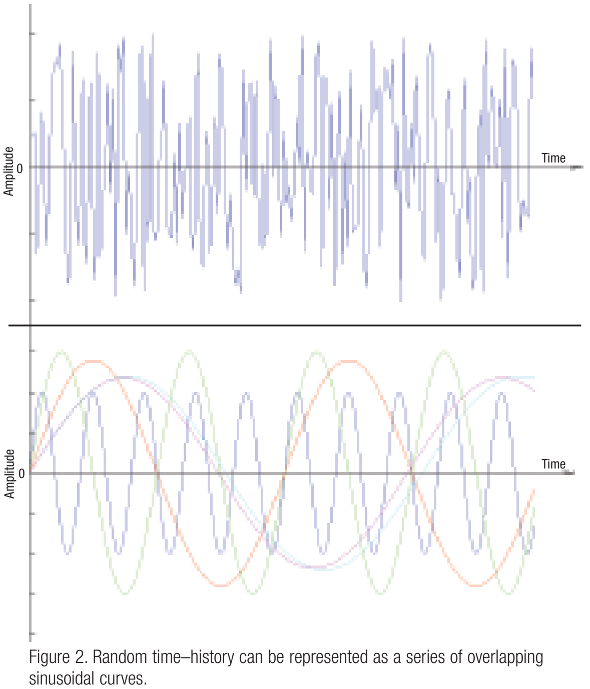

Figure 2. Random time–history can be represented as a series of overlapping sinusoidal curves.

An important aspect of such a statistical representation is that most random processes follow a Gaussian probability distribution. This distribution can be seen in a frequency-of-occurrence histogram (sometimes referred to as probability density function), which plots the number of times random acceleration peaks reached certain levels in small frequency segments called bins. The histogram shown in Figure 3 represents a random signal measured for 10,000 seconds and indicates that this random signal follows a classic bell-shaped Gaussian probability distribution.

Representing the random signals in this manner is sometimes called a zero-mean Gaussian process, since the mean value of the signals centers at zero of the histogram, as do the random signal responses, which are usually described in terms of standard deviation (or sigma value) of the distribution. Figure 3 shows how the Gaussian distribution relates to the magnitude of the acceleration levels expected for random vibration. The instantaneous acceleration will be between the $+1\sigma$ and the -1σ values 68.3 percent of the time. It will be between the $+2\sigma$ and the -2σ values 95.4 percent of the time. It will be between the $+3\sigma$ and the -3σ values 99.73 percent of the time. Note that the Gaussian probability distribution does not indicate the random signal's frequency content. That is the function of the power spectral density analysis.

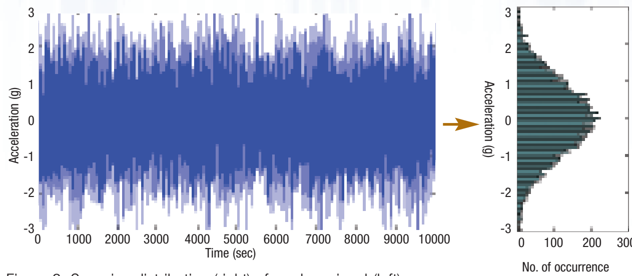

Figure 3. Gaussian distribution (right) of random signal (left)

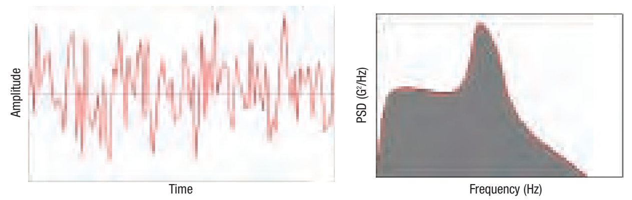

Figure 4. Random time–history (left), power special density (PSD) of a random time-history (right)

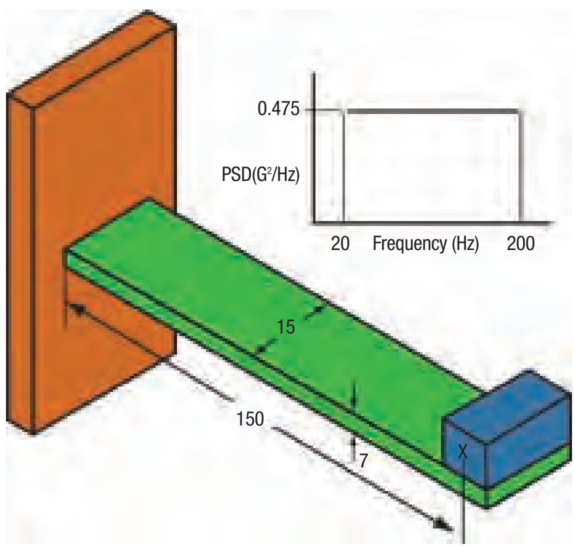

Figure 5. Problem sketch of aluminum beam with a weight at the tip undergoing white-noise random vibration

Power Spectral Density

The usual way to describe the severity of damage for random vibration is in terms of its power spectral density (PSD), a measure of a vibration signal's power intensity in the frequency domain. Looking at the time–history plot in Figure 4, it is not obvious how to evaluate the constantly changing acceleration amplitude. The way to evaluate is to determine the average value of all the amplitudes within a given frequency range. Although acceleration amplitude at a given frequency constantly changes, its average value tends to remain relatively constant. This powerful characteristic of the random process provides a tool to easily reproduce random signals using a vibration test system.

Random vibration analysis is usually performed over a large range of frequencies — from 20 to 2,000 Hz, for example. Such a study does not look at a specific frequency or amplitude at a specific moment in time but rather statistically looks at a structure's response to a given random vibration environment. Certainly, we want to know if there are any frequencies that cause a large random response at any natural frequencies, but mostly we want to know the overall response of the structure. The square root of the area under the PSD curve (grey area) in Figure 4 gives the root mean square (RMS) value of the acceleration, or Grms, which is a qualitative measure of intensity of vibration.

PSD Analysis Sample Problem

To illustrate how power spectral density analysis is used in calculating the fatigue life of a part undergoing random vibration, consider a cantilevered aluminum beam (Al 6061-T6 [E=68.9 GPa, γ=0.3]) that is 150 mm long by 15 mm wide by 7mm high, as shown in Figure 5. This system has an overall damping ratio of 5 percent. An instrument assembly of weight 2N is mounted on the tip of the beam, and its movement is restricted to only the vertical direction. The assembly must be capable of operating in a white-noise random vibration environment with an input PSD level of 0.475 g^2/Hz (from 20 to 200 Hz) for a period of 4.0 hours. The challenge is to determine the approximate dynamic stress and the expected fatigue life of the assembly.

Analysis of the assembly under this white-noise environment results in a bending stress contour plot shown in Figure 6, which shows a maximum 1-σ bending stress of 55.4 MPa (see accompanying table).

Standard Deviation	Bending Stress	Percentage of Occurrence
1σ stress	1x 55.4 = 55.4 MPa	68.3%
2σ stress	2x 55.4 = 110.8 MPa	27.1%
3σ stress	3x 55.4 = 166.2 MPa	4.33%

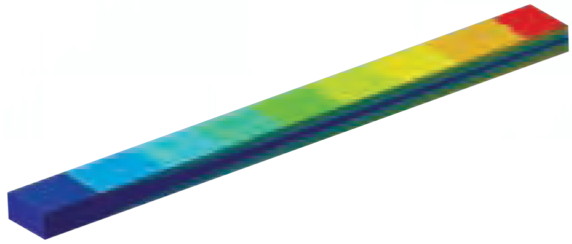
Figure 6. 1-σ bending stress distribution

Reponse Power Spectral Density (RPSD)

Figure 7 shows a response power spectral density plot (new in Workbench 12.0) of a node at root having maximum bending stress at the system's first natural frequency of ~56 Hz. The integration of the RPSD curve (the area under the curve) yields variance of bending stress. The square root of the variance is 1σ value of the bending stress.

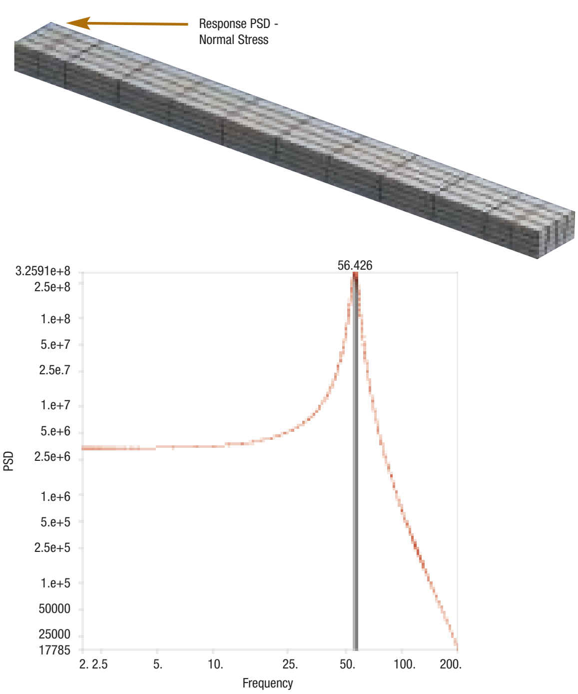

Figure 7. Response power spectral density of bending stress distribution for aluminum beam

Fatigue Analysis

For fatigue life calculation in the sample problem, root mean square (RMS) stress quantities are used in conjunction with the standard fatigue analysis procedure. The following procedure explains how to calculate the fatigue life using one of the most common approaches: the Three-Band Technique using Miner's Cumulative Damage Ratio [1].

The first step is to determine the number of stress cycles needed to produce a fatigue failure. When the root of the beam is connected to the other parts of the structure without any fillet, the computed alternating stress has to account for stress concentration effects. The stress concentration factor K can be used in the stress equation or in defining the slope b of the S-N fatigue curve for alternating stresses. The stress concentration should be used only once in either place. For this sample problem, a stress concentration factor K = 2 will be used in the S-N fatigue curve as shown in Figure 8, where slope b = 6.4.

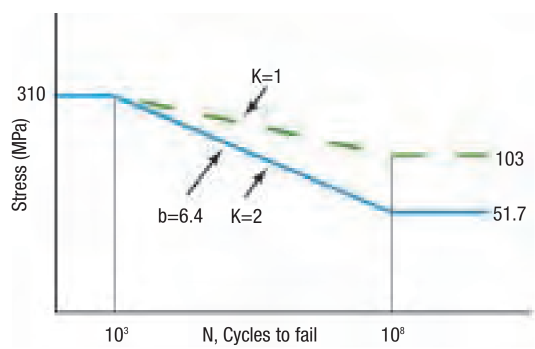

Figure 8. S-N curve for 6061-T6 aluminum beam with a stress concentration of 2

The approximate number of stress cycles N_1 required to produce a fatigue failure in the beam for the 1σ, 2σ and 3σ stresses can be obtained from the following equation:

$$N_1 = N_2\left(\frac{S_2}{S_1}\right)^b$$

where:

N_2 = 1000 (S_{1000} reference point)

S_2 = 310 MPa (stress to fail at S_{1000} reference point)

S_1 = 55.4 (1σ RMS stress)

b = 6.4 (slope of fatigue line with stress concentration K = 2)

The number of cycles to fail (N) under dynamic stress is calculated as follows:

$$1\sigma\ N_1 = (1000)\left(\frac{310}{1\times 55.4}\right)^{6.4} = 6.11\times 10^7$$

$$2\sigma\ N_2 = (1000)\left(\frac{310}{2\times 55.4}\right)^{6.4} = 7.24\times 10^5$$

$$3\sigma\ N_3 = (1000)\left(\frac{310}{3\times 55.4}\right)^{6.4} = 5.40\times 10^4$$

The actual number of fatigue cycles (n) accumulated during four hours of vibration testing can be obtained from the percent of time exposure for the 1σ, 2σ and 3σ values:

$$1\sigma\ n_1 = (56\,\text{cyc/sec}) \times (4\,\text{hr}) \times (60 \times 60\,\text{sec/hr}) \times (0.683) = 5.51 \times 10^5\ \text{cycles}$$

$$2\sigma\ n_2 = (56\,\text{cyc/sec}) \times (4\,\text{hr}) \times (60 \times 60\,\text{sec/hr}) \times (0.271) = 2.19 \times 10^5\ \text{cycles}$$

$$3\sigma\ n_3 = (56\,\text{cyc/sec}) \times (4\,\text{hr}) \times (60 \times 60\,\text{sec/hr}) \times (0.0433) = 3.49 \times 10^4\ \text{cycles}$$

Miner's Rule

Miner's cumulative fatigue damage ratio is based on the idea that every stress cycle uses up part of the fatigue life of a structure, whether the stress cycle is due to sinusoidal vibration, random vibration, thermal cycling, shock or acoustic noise.

Miner's fatigue damage cycle ratio calculation is as follows:

$$R_n = \sum_{i}^{n} \frac{n_i}{N_i} = \frac{5.51 \times 10^5}{6.11 \times 10^7} + \frac{2.19 \times 10^5}{7.24 \times 10^5} + \frac{3.49 \times 10^4}{5.40 \times 10^4} = 0.0090 + 0.3019 + 0.6462 = 0.9571$$

An examination of the above fatigue cycle ratio shows that the 1σ RMS level does very little damage even though it is in effect about 68.3 percent of the time. Most of the damage is generated by the 3σ level, even though it acts only about 4.33 percent of the time. The 3σ level generates more than two times as much damage as the 2σ level, which acts about 27.1 percent of the time.

The above fatigue cycle ratio shows that about 95.71 percent of the life of the structure is used up by the four-hour vibration test. This means that 4.29 percent of the life remains, with the expected life of the structure obtained from the following calculation:

Used life + remaining life = 4.0 hrs + [(4.0) x (0.0429)] = ~4.17 hrs

While fatigue life evaluation under a random process is highly complicated, Miner's Rule provides a reasonably good prediction. In the example, the safety factor of 2 calculated from structural stress values is not adequate to ensure fatigue life of the beam for the chosen environment. When it comes to design for manufacturing, it is recommended that the beam design be changed to provide a fatigue life of approximately 8 hours, amounting to a safety factor of 2 on the fatigue life. ■

Reference:

[1] Steinberg, D.S., "Vibration Analysis for Electronic Equipment," John Wiley & Sons Inc., 2000.

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